School Infrastructure Planning: Reducing Enrollment Pressure for 2021-2025

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Abstract

This paper addresses the overcrowding present in elementary and secondary schools in Surrey, British Columbia, Canada. The goal is to minimize the difference between a given school's capacity and its enrollment through the construction of new schools, expansion of existing schools, and the redrawing of catchment zones. This study covers the years 2021-2025, and uses forecasts of population based off of government census tract data using linear regression. The issue is formulated as an integer programming model where the binary decision variables indicate whether or not to build a school at a predetermined location or expand a specific school, and three solutions are presented for three levels of capital budget. Afterwards, the catchment zones are reworked based off of one of the solutions.

1 Introduction

Limited school capacities combined with explosive growth in the population of Surrey, B.C. has led to a serious overcrowding problem in the city's public school system. The aim of this paper is to formulate a model to reduce the overcrowding faced by Surrey in both elementary and secondary schools. Overcrowding in schools forces teachers to split their time and attention among too many students.

Historically, portable classrooms have often been used as substitutes for actual classrooms. However, portable classrooms are frequently criticized for presenting mold, moisture, and comfort issues. The number of portables in Surrey's inventory has fluctuated over the years for a variety of reasons, including overcrowding and construction, but portable classrooms are not a sustainable solution to the overcrowding issue and are not a substitute for proper school buildings.

School locating models are typically formulated as integer programs where the objective is to minimize cost or maximize accessibility of spaces. Antunes and Peeters (2001) [5] studied school location planning where the objective was to find locations for schools to be built. The model they used was the “dynamic modular capacitated facility location problem" (DMCFLP) model, and the objective was to minimize the cost of all new facilities while satisfying a certain level of demand. Caro et al. (2004) [1] used integer programming to redraw the catchments of schools, minimizing the walking distance for every block by GIS Tools. Unfortunately, census tract population was the only data available to us, and catchment zones in Surrey are small enough that walking distance may not be a main factor in drawing catchment boundaries.

The Surrey school district has proposed a five year capital plan for 2016 to 2020 which details plans to build new schools and expand several others. The aim of this paper is to provide a five year plan for 2021 to 2025 that is feasible and effective in reducing overcrowdedness.

1.1 Scope of the project

From 2005 to 2015, the total population of Surrey increased from 418,330 to 516,650, and the total population is projected to approach 630,000 by 2025 [4]. This will amplify the overcrowding concerns Surrey, and is the reason why the school district and the provincial government are committing $217 million over 3 years [20] to keep up with increasing demand. Currently, the public school system has a total enrollment of 27,300 students, and a total capacity of 24,405. This means that the system is approximately 12% overcapacity with 2,895 students in excess.

This paper takes into account most public elementary and secondary schools (subject to some exclusions) in Surrey. There are 96 elementary schools, 19 secondary schools, and 101 census tracts. Surrey has an area of 316.4km^2, making it the third largest city in British Columbia [12]. It spans the area between 120 Street and 196 Street, and between 0 Avenue and 117 Avenue.

2 Preliminaries

2.1 Data collection

Surrey population data was collected from the 2001, 2006, 2011, and 2016 Census Program directed by Statistics Canada. The data retrieved from the 2006 and 2011 censuses breaks down the elementary and secondary-aged populations, that is, the age groups 5-9, 10-14, and 15-19 in each census tract. People aged 18 and 19 are considered to have graduated secondary school, and have thus been excluded from the data. It is assumed that the population is uniformly distributed for the 5 years in an age group, so 60% of the 15-19 population is assumed to be the entire 15-17 population. Unfortunately, only the total population per tract is available for the years 2001 and 2016. To obtain age group estimates for these years, the percentage that each age group makes up of the total for 2006 and 2011 was calculated, the two values were averaged, and this percentage was multiplied by the total population per tract for 2001 and 2016.

Capital budget, cost of building a new school, and cost of expansion were collected from a 2017 news release [18], 2014 capital update [16], and a 2016/17 five-year capital plan [17] developed by the B.C. Ministry of Education. Note that the capital budget estimate is the sum for the five years of this study. The predicted budget is in the $250M-300M range. The capital update shows enrollment growth, current capital projects and plans, current land and capital assets, and typical capital development timelines and costs. The five-year capital plan outlines expansions and various other building enhancements with estimated costs for some schools. These numbers were all used (as well as an estimated inflation of 1.49%) when producing estimates for costs of new schools and expansions.

Current school locations and their catchment zones are displayed in the 2016/17 Surrey school catchment boundary map [19] developed by the B.C. Ministry Of Education. The map consists of secondary and elementary school catchment boundaries, in blue and red respectively, overlaid onto a map of Surrey (see Figure 1).

While the population in each of the catchment boundaries was desired, no such data was available. To capture the population in a catchment zone, the sum of the populations of the census tracts inside the zone was calculated, taking into account the fact that some census tracts are split between catchment zones. This was done for all 96 elementary zones, which could then be grouped into secondary catchment zones.

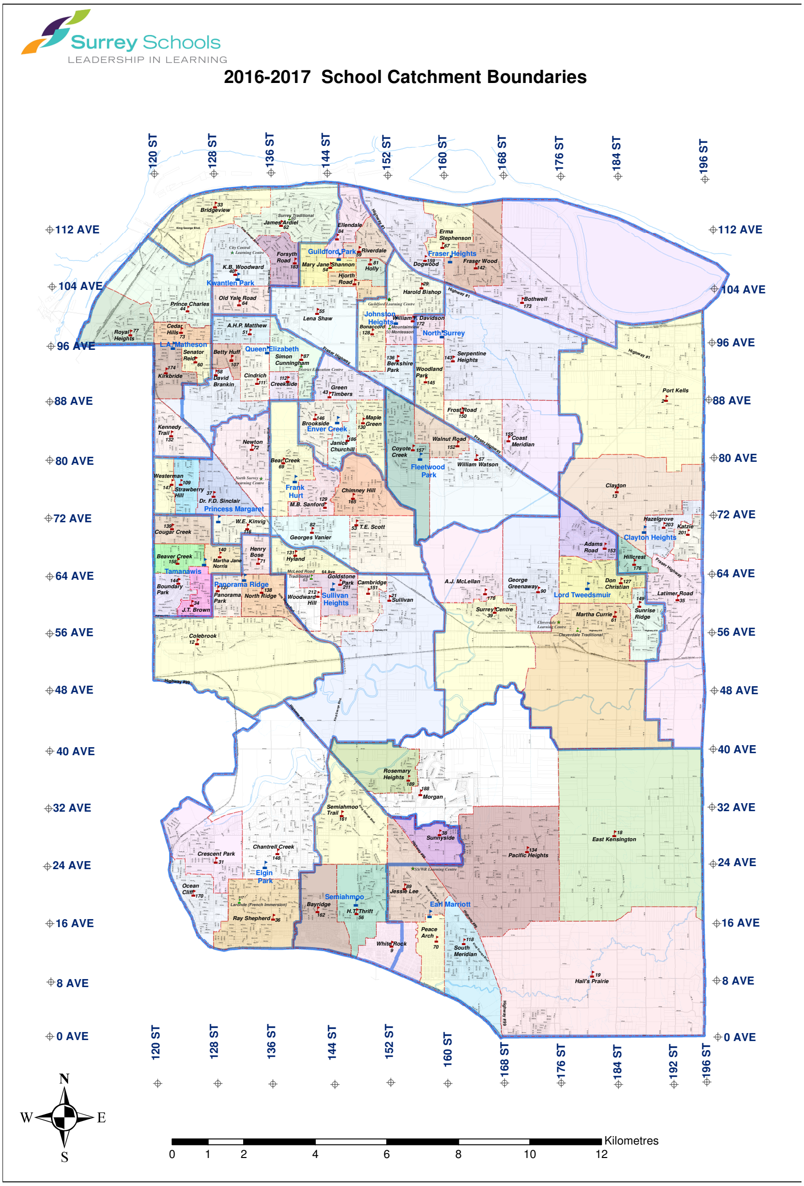

Figure 1: 2016/17 catchment boundary map.

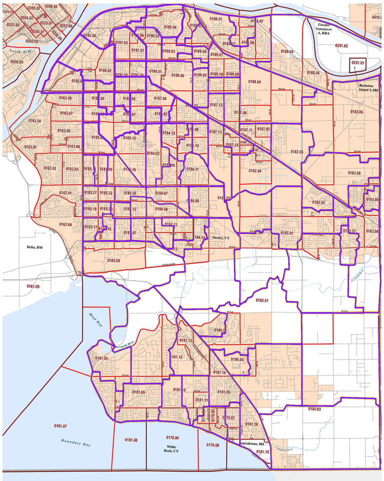

Figure 2: Elementary catchments on the census map

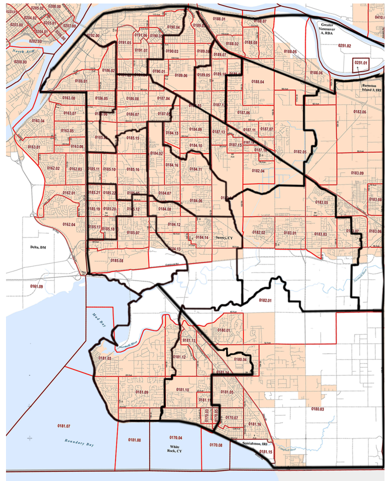

Figure 3: Secondary catchments on the census map

2.2 Defining adjacency: elementary and secondary catchment zones

The 2016/17 Surrey school catchment boundary map was used to determine the adjacency of schools. For secondary schools, any zone *i* that shares a border (including diagonally) with a zone *j* is said to be adjacent to *j*, and vice versa. The corresponding entries a_{ij} and a_{ji} of the adjacency matrix are then both equal to 1. The same holds true for elementary schools, only for adjacency matrix v_{kl}.

2.3 Assumptions

For the sake of simplicity and due to lack of resources, several assumptions have been made. First, it is assumed that population growth follows a linear trend. This is a reasonable approximation in the short term. In the long run, one might expect population to follow a logistic curve, but modeling that would require a more refined understanding of the population behavior, which is difficult to obtain. Also, it is assumed that students attend school in their catchment zone. For the most part this is true, though some small amount of students apply to go to schools outside their catchment due to program availability or sports teams. Finally, it is assumed that a school that has been recently expanded is ineligible for further expansion, as there were no such cases in the historical capital projects [8].

2.4 Exclusions

For the same reasons that the previous assumptions were made, some aspects of the school system were excluded from the model. Private schools, learning centres, and home-schooled children were all excluded due to lack of available data and being a relatively small part of the system. For example, Southridge School, a K-12 private school in South Surrey, has an enrollment of only 680 students [10].

3 Methodology

3.1 Projected population

Estimates were derived for the elementary and secondary-aged populations in each census tract using data from the 2001, 2006, 2011, and 2016 censuses. The historical data is a good fit for a linear model, and this trend is assumed to hold for 2025. The overall population was projected based on a linear regression model in R.

The projected elementary and secondary populations in the tracts provide an estimate of the student population in each catchment. The total expected elementary and secondary students in 2025 are 66,081 and 43,855 respectively. Historically, elementary and secondary-aged populations have made up 10% and 7% of the total population. This holds true for the 2025 population projected by the City of Surrey [4].

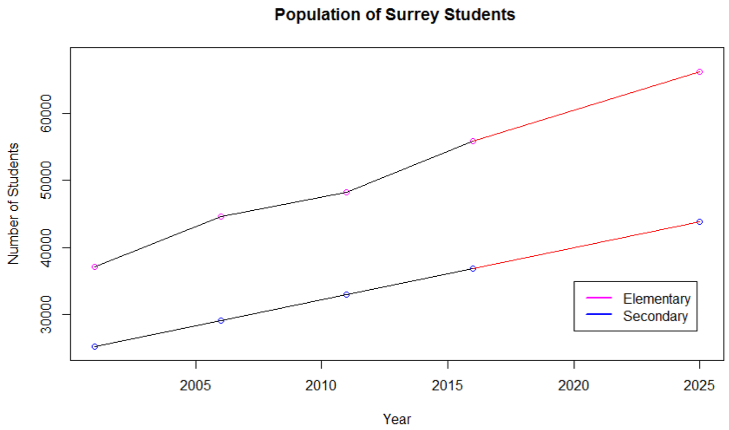

Figure 4: Historical and projected student population.

3.2 Forecasted capital development and expansion costs

When building a school, the three main factors to consider are its capacity, lot size, and the cost of construction and labor. Cost of land per hectare and cost of construction per hectare were based off of the 2014 capital budget [18]. Cost of construction and labor cost were estimated separately for elementary and secondary schools. Cost of land was estimated to be $2.4 million/hectare, and cost of construction was estimated to be $5.58 million per hectare for elementary schools, and $6.84 million per hectare for secondary schools.

The area standards provided by the B.C. Ministry of Education [6] served as guidelines for required lot size by building capacity. However, it can be observed from the five-year plan that the purchased lot size is always larger than the required lot size. Factors of 1.06 for elementary schools and 1.21 for secondary schools were used when determining the lot size to purchase for a school of given capacity.

An average rate of inflation of 1.49% was calculated based off of historical inflation, and used to predict future costs of land and construction for both elementary and secondary schools. These were calculated as roughly $3.1 million per 100 spaces for elementary school, and $3.21 million per 100 spaces for secondary school (see Tables 1 and 2).

$$fe_{cap} = (1.06 * r_{cap}) * (c_e + l) * (1 + i)^n$$

$$fs_{cap} = (1.21 * r_{cap}) * (c_s + l) * (1 + i)^n$$

fe_{cap} = future cost of a new elementary school of size-cap
fs_{cap} = future cost of a new secondary school of size-cap
r_{cap} = required lot size to build a cap-sized school
cap = capacity
c_e = elementary school construction cost per hectare
c_s = secondary school construction cost per hectare
l = cost of site per hectare
i = inflation rate
n = number of years to bring forward to 2025

Capacity	Price (Million $)
1000	61.3
1100	65
1200	68.6
1300	71.1
1400	74.8
1500	77.2
1600	80.9
1700	83.3

Table 1: Estimated secondary school construction costs.

Capacity	Price (Million $)
200	13.9
250	14.8
300	16.7
350	17.6
400	21.3
450	23.2
500	25.1
550	26
600	27.8

Table 2: Estimated elementary school construction costs.

3.3 Integer programming model

The objective function maximizes the number of seats created through expansion and construction of new schools. This is equivalent to minimizing the difference between the number of school-aged individuals and the total capacity of the school in each zone that is over capacity. There are 3 sets of variables for both elementary and secondary schools: a decision variable for whether or not a new elementary or secondary school should be built within a zone, another decision variable for how much an existing school should be expanded, and a last variable for determining the number of students that should be transferred from adjacent zones to new schools.

There are 6 sets of indices: 2 sets comprised of the secondary and elementary school catchment zones, another 2 for the subsets of the previous ones but only including overcapacity zones, and the last 2 are the ranges of capacity for new schools.

3.4 Parameters

J = the set of all secondary school zones
L = the set of all elementary school zones
α = upper bound on capacity for secondary schools
β = upper bound on capacity for elementary schools

For *i,j* $\in$ *J*, let
S_j = the number of students living in catchment zone j,
c_j = the capacity of secondary school j,
σ_j = the maximum capacity of secondary school j,
ϕ = the cost per 100 seats of expanding secondary school,
γ_g = the capacity of new secondary school with size g (see Table 1),
a_{ij} = 1 if i,j are adjacent or the same,
θ_g = the cost of building a secondary school with size g

For *l,k* $\in$ *L*, let
E_k = the number of students in catchment zone k,
d_k = the capacity of elementary school k,
σ_l = the maximum capacity of elementary school l,
ω = the cost per 100 seats of expanding elementary school,
δ_h = the capacity of new elementary school with size h (see Table 2),
b_{kl} = 1 if k,l are adjacent or the same,
ψ_h = the cost of building an elementary school of size h

3.5 Variables

I = {i $\in$ J $|$ secondary school i is over capacity}
K = {k $\in$ L $|$ elementary school k is over capacity}

For $i \in I, j \in J$,
w_i = number of spaces (in hundred seats) added to secondary school i by expansion,
y_{jg} = 1 if a new school is built in zone j; 0 otherwise,
u_{ij} = number of students transferring from zone i to a new school in zone j

For $k \in K, l \in L$,
x_k = number of spaces (in hundred seats) added to elementary school k by expansion,
z_{lh} = 1 if a new elementary school is built in zone l; 0 otherwise,
v_{kl} = number of students transferring from zone k to a new school in zone l

3.6 Objective function

The goal of the objective function is to maximize the number of seats created through school expansion and construction. A 10% penalty is added on transferring from different zones so that students in the same zone have the highest priority to get into a new school. 10% was chosen because it is small enough to allow outside transfers, but large enough to prioritize transfers within a zone. Attempts to solve the model with a larger penalty resulted in the elimination of outside transfers.

$$\max \quad 100\sum_{i\in I} w_i + \sum_{i,j\in I} u_{ij} - 0.1\sum_{i,j\in I|i\neq j} u_{ij} + 100\sum_{k\in K} x_k + \sum_{k,l\in K} v_{kl} - 0.1\sum_{k,l\in K|k\neq l} v_{kl}$$

3.7 Constraints

The total number of transfers out of a zone should be no more than the number of excess students in the zone.

$$\sum_j u_{ij} \leq S_i - c_i - 100w_i \qquad \forall i$$

$$\sum_l v_{kl} \leq E_k - d_k - 100x_k \qquad \forall k$$

A school should not expand beyond its maximum capacity.

$$100w_i \leq \sigma_i - c_i$$

$$100x_k \leq \tau_k - d_k$$

Two schools must be adjacent to each other in order to transfer students.

$$u_{ij} \leq a_{ij}\sum_{g\in G}\gamma_g y_{jg} \qquad \forall i,j$$

$$v_{kl} \leq b_{kl}\sum_{h\in H}\delta_h z_{lh} \qquad \forall k,l$$

The total number of students transferring to a new school must be at most the capacity of the new school.

$$\sum_{i\in I} u_{ij} \leq \sum_{g\in G}\gamma_g y_{jg} \qquad \forall j$$

$$\sum_{k\in K} v_{kl} \leq \sum_{h\in H}\delta_h z_{lh} \qquad \forall l$$

More students must transfer from within a zone than from any other nearby zone.

$$u_{ij} \leq u_{jj} \qquad \forall i, j \in I, \quad i \neq j$$

$$v_{kl} \leq v_{kk} \qquad \forall k, l \in K, \quad k \neq l$$

Each secondary school must be at most α% overcapacity, and each elementary school must be at most β% overcapacity.

$$S_i - \sum_j u_{ij} \leq (1+\alpha)(c_i + 100w_i) \qquad \forall i$$

$$E_k - \sum_l v_{kl} \leq (1+\beta)(d_k + 100x_k) \qquad \forall k$$

Total expenditure must be less than or equal to the capital budget.

$$\sum_{i \in I} \phi w_i + \sum_{k \in K} \lambda x_k + \sum_{j \in I g \in G} \theta_g y_{jg} + \sum_{l \in K, h \in H} \psi_h z_{lh} \leq Budget$$

We ran the model for three potential capital budgets: \$250M, \$300M and \$350M. The maximum overcapacity percentages α and β are parameters, but different pairs of α and β give very different optimal solutions. A large α makes the model prioritize elementary schools, and vice versa. It is important to pick a pair of α and β that fairly weights both elementary and secondary schools. Fairness is quite subjective, but we decided to minimize the difference between the average overcapacity percentage for elementary and secondary schools.

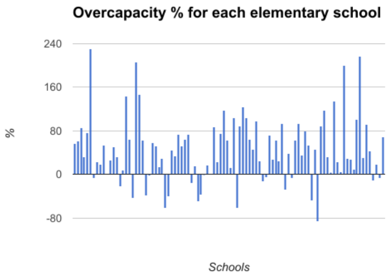

Figure 5: Overcapacity percentage for elementary schools.

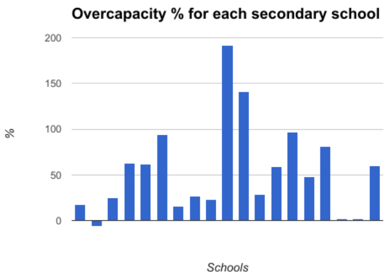

Figure 6: Overcapacity percentage for secondary schools.

α and β were used to force the model to solve for the extreme cases first (see Figures 5 and 6). Minimizing α and β is equivalent to minimizing the highest overcapacity percentage of all schools. However, the solution space is not convex - it ends up having two pairs of α and β (either a emphasizing elementary schools or secondary schools) which satisfy the fairness criteria mentioned in the previous paragraph. Whenever there is a tie between two pairs, the one with a higher objective function value is preferred.

A solution to this model indicates which schools to expand and where to locate new schools. Some school zones are very large, so it is important to determine where the new school should be located within a zone. Since the model also determines the number of transfers from different school zones, these numbers play a part in locating the new school on the map. Of course, in order to set an appropriate location for a school, it is necessary to modify the existing catchment zones. However, because population is not evenly spread and the catchment boundaries are irregular, to do so precisely would require knowledge of the population distribution in each zone. The next best option is to use the census tract projections and observe the geography of the tracts (some places are clearly undeveloped and thus uninhabited) to redraw the catchments. There are many possible ways of dividing the catchments and the one presented in this paper is merely a suggestion. In reality, catchment zones affect the values of homes and have both political and social ramifications. This means that the government would need to consult the public and do extensive planning before actually committing to any changes in catchment zones.

4 Results

4.1 $250M capital budget

The model produces a solution of 2 secondary school expansions, 12 elementary school expansions, construction of 1 new secondary school, and construction of 8 new elementary schools. The results are summarized in Table 3. The average overcapacity % are approximately equal, even though the solution has reduced more crowdedness in elementary schools.

	Secondary	Elementary
Cost ($M)	71.7	178.1
Seats Created	1400	4150
Overcapacity	12443	18011
Maximum Overcapacity %	98	85
Average Overcapacity %	46	46

Table 3: Solution for $250M budget.

4.2 $300M capital budget

The model produces a solution of 2 secondary school expansions, 15 elementary school expansions, construction of 1 new secondary school, and construction of 10 new elementary schools. The results are summarized in Table 4. Much of this solution is the same as the $250M solution, with the extra $50M being spent on additional spaces for elementary schools.

	Secondary	Elementary
Cost ($M)	71.7	228.1
Seats Created	1400	5400
Overcapacity	12443	16761
Maximum Overcapacity %	98	71
Average Overcapacity %	46	41

Table 4: Solution for $300M budget.

4.3 $350M capital budget

The model produces a solution of 2 secondary school expansions, 12 elementary school expansions, construction of 2 new secondary school, and construction of 10 new elementary schools. The results are summarized in Table 5. This solution is fairly different from the previous ones. α is minimized to a more reasonable 87% while β increases by 2%. This pair of α and β has the yields the most equal average overcapacity % for elementary and secondary schools.

	Secondary	Elementary
Cost ($M)	130.4	219.6
Seats Created	2300	5200
Overcapacity	11543	16961
Maximum Overcapacity %	87	73
Average Overcapacity %	41	42

Table 5: Solution for $300M budget.

4.4 Drawing new catchments

Redrawing catchments was done by hand, where we considered the population of each zone by census tract and tried to minimize changes to the original catchments. We redrew catchments only for the $250M capital budget scenario (see Appendix D), as the process is somewhat subjective and our proposal is one possibility of many. For the new secondary school in the Frank Hurt zone (see Figure 5), we noticed that the new school had a capacity of 1,000 students, approximately equal to the population of the Chimney Hills. Thus, we decided to have the new school catchment encompass the Chimney Hills area entirely. Also, the solution dictates a transfer of 750 students from Beaver Creek elementary to the 2 new schools in its zone, so we drew two sections to match the proportion of students they will receive. One potential improvement to the model would be to allow newly constructed schools to be overcapacity. This would reduce the strain on nearby existing schools.

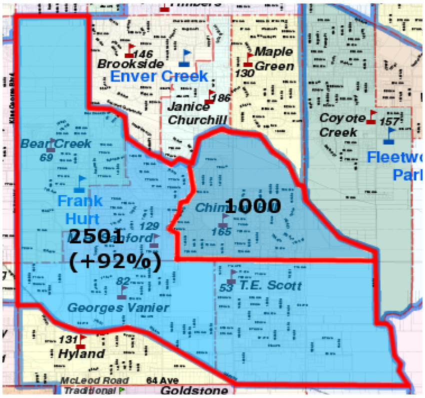

Figure 7: Revised catchments for Frank Hurt.

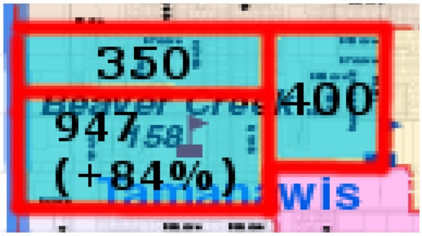

Figure 8: Revised catchments for Beaver Creek.

4.5 Computations

There are 6,861 variables and 12,178 constraints. The integer program was solved using AMPL on a Lenovo IdeaPad Y50-70 with an Intel i7-4710HQ CPU @ 2.50GHz and 8 GB of RAM. The first run was solved in 3.187 seconds, the second in 11.203 seconds, and the third in 24.641 seconds.

5 Summary

The above analysis provides a suggested course of action for three levels of capital budget in facing the level of overcrowding predicted for 2025. While the proposed solutions do much to improve the issue, the issue is far from being solved. The public school system remains overcapacity in all cases by more than 25,000 students. Lack of budget combined with the immense cost of capital projects means that the infrastructure can not be upgraded fast enough to keep up with population growth.

The model provides optimal solutions given the constraints, but the conclusion to be drawn is that the constraints are too binding to even approach solving overcrowdedness. The School District simply needs more budget or significantly lower costs of capital projects in order to properly address the issue.

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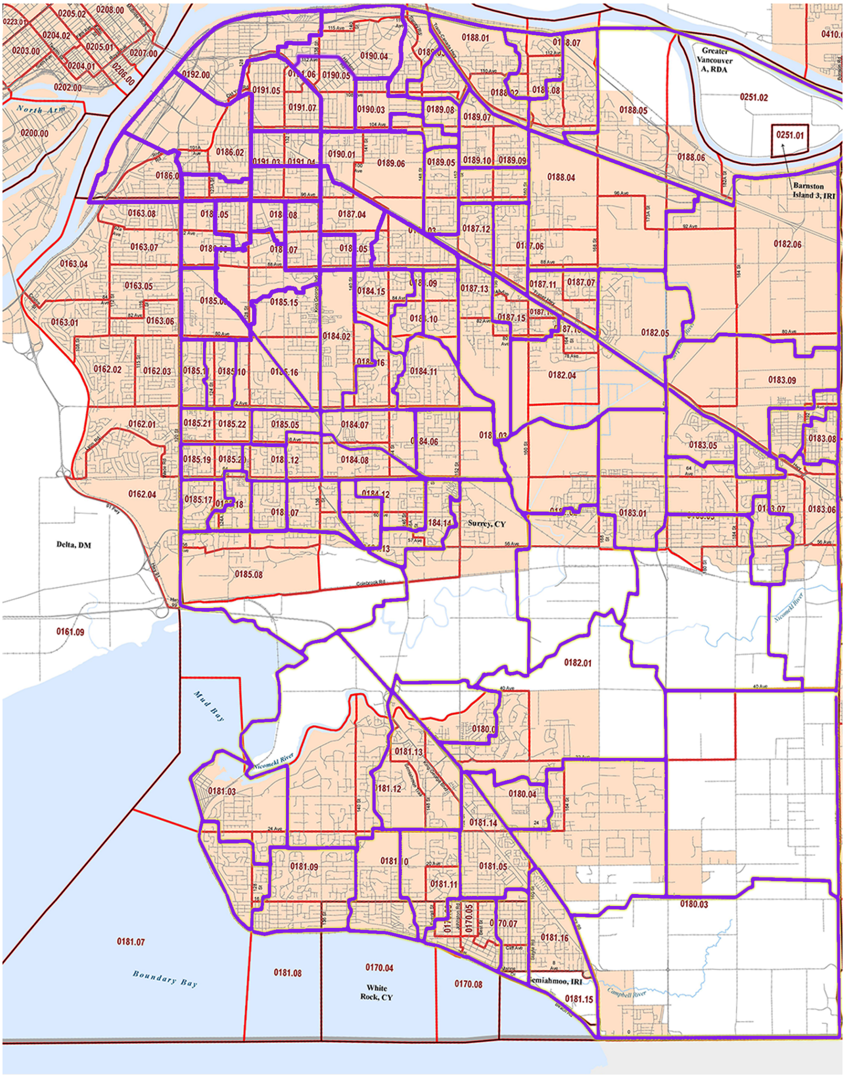
A Figures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Barnston Island 3, IRI
Delta, DM
Surrey, CY
Mud Bay
Nicomekl River
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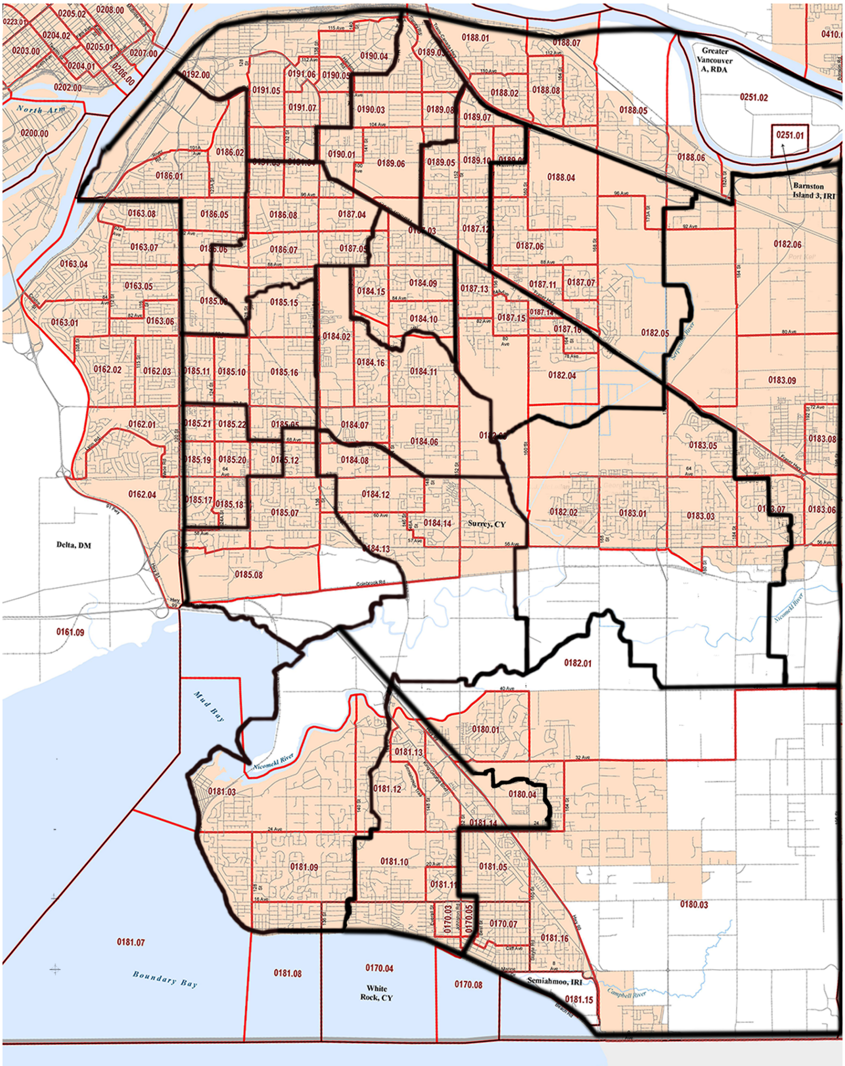
0180.04
0180.03
0170.05
0170.04
0170.08


Greater Vancouver A, RDA
0251.02
0251.01
Barnston Island 3, IRI
North Arm
Delta, DM
Surrey, CY
Mud Bay
Nicomekl River
Serpentine River
Boundary Bay
White Rock, CY
Semiahmoo, IRI
Campbell River
0188.01
0188.07
0188.05
0188.04
0188.06
0182.06
0182.05
0182.04
0182.02
0182.01
0183.09
0183.08
0183.05
0183.01
0183.03
0183.06
0180.01
0180.03
0180.04
0181.03
0181.07
0181.08
0181.09
0181.10
0181.12
0181.13
0181.14
0181.15
0181.16
0181.05
0170.04
0170.07
0170.08
0170.03
0170.05
0162.01
0162.02
0162.03
0162.04
0163.01
0163.04
0163.05
0163.06
0163.07
0163.08
0161.09
0200.00
0202.00
0203.00
0192.00
0185.08
0184.12
0184.13
0184.14

B R Code

```
period = 2025 # Prediction year

# data reading
d11 = read.csv("MATH402␣Data␣–␣2011.csv",header = TRUE)[2:6]
d06 = read.csv("MATH402␣Data␣–␣2006.csv",header = TRUE)[2:6]
d16 = read.csv("MATH402␣Data␣–␣2016.csv",header = TRUE)[2:6]
d01 = read.csv("MATH402␣Data␣–␣2001.csv",header = TRUE)[2:6]
# prediction
year = matrix(c(2001,2006,2011,2016) , ncol = 1)
prediction = matrix(c(1) , ncol = 4 , nrow = dim(d11)[1])
for (i in 1:dim(d11)[1]){
  for (j in 1:4){
    r = rbind(d01[i,j] , d06[i,j] , d11[i,j] , d16[i,j])
    fit = lm(r ~ year)
    pop = fit$coefficients[1] + fit$coefficients[2] * period
    prediction[i,j] = round(pop)
  }}
colnames(prediction) = c("0–4" , "5–9" , "10–14" , "15–19")
write.csv(prediction , file = "prediction.csv")
```

C Model

$$
\begin{aligned}
\max \quad & 100\sum_{i\in I} w_i + \sum_{i,j\in I} u_{ij} - 0.1 \sum_{i,j\in I|i\neq j} u_{ij} + 100\sum_{k\in K} x_k + \sum_{k,l\in K} v_{kl} - 0.1 \sum_{k,l\in K|k\neq l} v_{kl} \\
\text{s.t.} \quad & \sum_j u_{ij} \le S_i - c_i - 100w_i \qquad \forall i \\
& \sum_l v_{kl} \le E_k - d_k - 100x_k \qquad \forall k \\
& 100w_i \le \sigma_i - c_i \\
& 100x_k \le \tau_k - d_k \\
& u_{ij} \le a_{ij} \sum_{g\in G} \gamma_g y_{jg} \qquad \forall i, j \\
& v_{kl} \le b_{kl} \sum_{h\in H} \delta_h z_{lh} \qquad \forall k, l \\
& \sum_{i\in I} u_{ij} \le \sum_{g\in G} \gamma_g y_{jg} \qquad \forall j \\
& \sum_{k\in K} v_{kl} \le \sum_{h\in H} \delta_h z_{lh} \qquad \forall l \\
& u_{ij} \le u_{jj} \qquad \forall i, j \in I, \quad i \neq j \\
& v_{kl} \le v_{kk} \qquad \forall k, l \in K, \quad k \neq l \\
& S_i - \sum_j u_{ij} \le (1+\alpha)(c_i + 100w_i) \qquad \forall i \\
& E_k - \sum_l v_{kl} \le (1+\beta)(d_k + 100x_k) \qquad \forall k \\
& \sum_{i\in I} \phi w_i + \sum_{k\in K} \lambda x_k + \sum_{j\in I g\in G} \theta_g y_{jg} + \sum_{l\in K, h\in H} \psi_h z_{lh} \le Budget
\end{aligned}
$$

D Redrawn Catchments

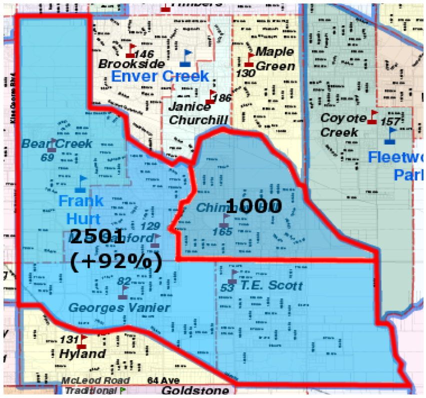


Figure 9: Revised catchments for Frank Hurt.

Figure 10: Revised catchments for Beaver Creek.

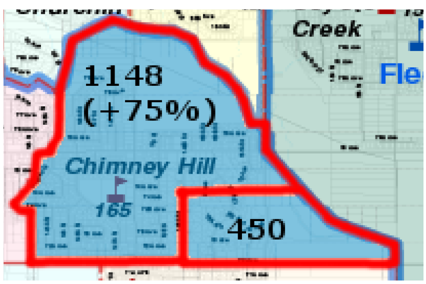


Figure 11: Revised catchments for Chimney Hills.

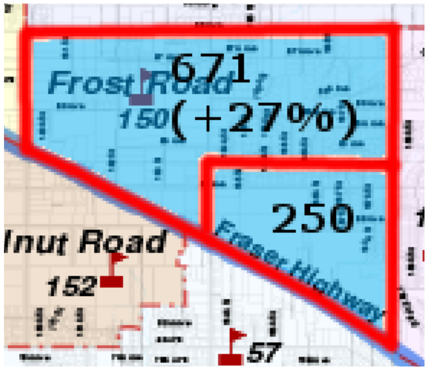


Figure 12: Revised catchments for Frost Road.

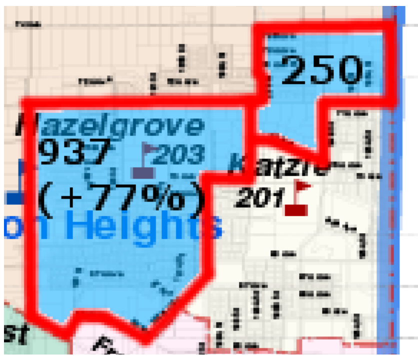


Figure 13: Revised catchments for Hazelgrove.

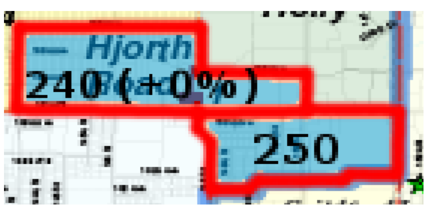


Figure 14: Revised catchments for Hjorth Road.

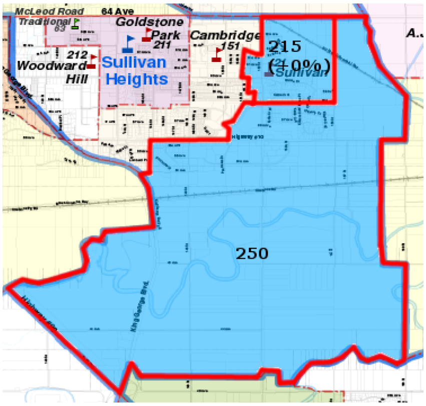


Figure 15: Revised catchments for Sullivan.

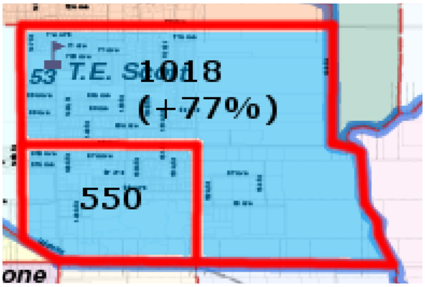


Figure 16: Revised catchments for T.E. Scott.

E Data

Data and adjacency matrices available at:
https://docs.google.com/spreadsheets/d/1PMcaRw04hOlK9OgWFC5o4\yrty6vMjxDCm2tkdL5I5v4/edit?usp=sharing