Exam Scheduling Problem

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1 Introduction

We introduce the exam scheduling problem where we wish to schedule exams for thousands of students over just two weeks. The desire to formulate a model to schedule exams comes from the abundance of students, like ourselves, that often complain that the final exams are scheduled too close together. We have found out that many students may have multiple exams on the same day, which can be very stressful for students. There are also extreme circumstances where students are scheduled to have two exams during the same time. Students often cannot perform to their best ability when faced with a stressful exam schedule. Therefore, we wish to reduce the stress of students during exam time by maximizing their study time between scheduled exams through our model.

2 Background

Exams in SFU are typically scheduled shortly after the last day of classes over roughly two weeks. The exams take place in three hour intervals starting at 08:30, 12:00, 15:30, and 19:00. In the Spring 2018 semester, the exam period is scheduled for 12 days, starting on Apr. 12 till Apr. 23, and will consist of the four time slots per day as described above. This results in a total of 48 time slots during the exam period. We have decided not to alter the set exam time slots because fitting four 3 hour exams over a course of day is already tight. It is not a fair option to decrease the length of the time of exams or force the exams to start earlier than 08:30 on a respective day.

3 Data

Scheduling exams for all the students enrolled in Simon Fraser University over a couple of weeks is a very big task. Simon Fraser University has both undergraduate and graduate students enrolled in the school. As the majority of students are undergraduate students, we focus on undergraduate courses that have exams. We narrow down the topic further to address the problem for undergraduate students enrolled in the *Faculty of Science* of *Simon Fraser University (SFU)*, since this is the faculty of most interest to us.

The data set used to solve the problem is provided by *Liny Chan, Analyst at SFU.* Orignally, the data set includes all Faculty of Science students and the respective courses they are registered in the Spring 2018 semester. This results in a total of 3402 students and 759 different courses taken amongst them. The problem is simplified in order to approach it in an effective manner. From the resulting data sheet, we have reduced the data down to 199 courses to only account for Faculty of Science courses, and courses that actually have final exams.

4 General Assumptions

SFU currently has 3 campuses (Burnaby campus, Surrey campus, and a Vancouver campus) located in the lower mainland. Although all three campuses could potentially hold exams, we simplify our model further by assuming all of the exams are held in the Burnaby Campus. This is a reasonable assumption as Burnaby is the main campus for SFU. As a result, a large proportion of classes are held in the Burnaby campus.

The model is also made under the assumption that different class sections for a course (for example D100, E100) are not taken into account, since students under the same course, regardless of class section, tend to have same exam times. The addition of these class section under our previous assumption (that all exams are held in Burnaby), is quite redundant.

5 Model Constraints

Decision Variables:

We introduce 199 integer decision variables for each of the courses in our dataset. We let t_i denote the time slot scheduled for course i, i = {ACMA320, ACMA340, ACMA455, ACMA470, BISC101,..., STAT475}. Note the set i is sorted under the alphabetical naming of the course and numerical number of the course. For example, t_{199} corresponds to the time slot scheduled for the final exam of STAT475.

$$t_i = \text{time slot scheduled for course i, where } t_i \text{ are integer}$$

Penalty:

We incorporate a penalty system to penalize certain circumstances of the resulting exam schedules. It is fair to scale the penalization of courses that take place in the exact same time slot (conflict), courses with exams on the same day, and multiple exams within less than 4 exam time slots away from each other. For example, if course i and j are scheduled within two time slots of each other, the penalty will be -50. Therefore, we introduce a scaled penalty variable p_{ij}.

$$p_{ij} = \begin{cases} -100000, & \text{if course i and j conflict} \\ -50, & \text{if distance between course i and j} < 2 \text{ time slots} \\ -30, & \text{if distance between course i and j} < 3 \text{ time slots} \\ -1, & \text{if distance between course i and j} < 4 \text{ time slots} \end{cases}$$

Matrices:

We also generate two separate matrices noted as C_{ij} and M_{ij} to take note of conflicts from the dataset.

$$C_{ij} = \begin{cases} 1, & \text{if course i and j are taken together} \\ 0, & \text{otherwise} \end{cases}$$

$$M_{ij} = \begin{cases} 0, & \text{if course i and j that are combinations have a conflict} \\ 1, & \text{otherwise} \end{cases}$$

Objective Function:

The objective function is designed to maximize the study time for Faculty of Science students registered in courses. "Maximizing study time" in our case refers to maximizing the times in between exams. In a sense, we are maximizing the distance between combination of course i and course j. The objective function takes into account the penalty p_{ij} of combination of course i and j. Therefore, our objective function is as follows:

$$\text{Maximize} \sum_{i=1}^{199} \sum_{j=1}^{199} d_{ij} + p_{ij}$$

$$\text{Where } d_{ij} = |t_i - t_j|$$

Constraints:

With a limit of 48 exam time slots within the 12 day exam period, the number of time slots, t is constrained:

$$1 \leq t \leq 48$$

In order to maximize the number of exams scheduled with no exam conflicts, the following constraint is added:

$$\text{Maximize} \sum_{i=1}^{199} \sum_{j=1}^{199} M_{ij}$$

$$\text{For } M_{ij} \leq 39601$$

$$\text{Where } 39601 = 199 \times 199$$

Let set E_k be the number of exams per day k, for k = 1,2,3,...,12. The number of science exams scheduled per day is constrained as following based on previous exam schedules (see reference):

$$E_k \leq 40$$

For k = 1,2,3,...,12

Let A_s be the average number of rooms used for each exam slot s = 1,2,3,4, corresponding to exams starting times: 8:30, 12:00, 15:30 and 19:00 respectively, for all 12 days of exams. These constraints are based on Spring 2018 exam schedule room uses (see reference).

$$A_1 \leq 7$$

$$A_2 \leq 6$$

$$A_3 \leq 6$$

$$A_4 \leq 3$$

6 Results

The maximized objective function value is -8,165,820.

The maximized value for constraint:

$$\sum_{i=1}^{199} \sum_{j=1}^{199} M_{ij} = 39{,}519$$

The decision variables t_i, the time slots assigned to each course i, is displayed in the tables below:

	Start Times			
Date	8:30	12:00	15:30	19:00
April 12	CHEM 283	CHEM 381 CHEM 432 EASC 201 EASC 302 EASC 313 PHYS 120	PHYS 100	BPK 420 CMPT 411
April 13	EASC 103 HSCI 321 PHYS 102 STAT 380	BPK 482 BUEC 33 CHEM 462 EASC 108 MATH 310 STAT 201	BPK 141 BPK 444 HSCI 211 MATH 443 STAT 350	BPK 312 BPK 301 CMPT 379 HSCI 324 MATH 154
April 14	BPK 110 BPK 205 BPK 443 CHEM 340 CMPT 218 EVSC 100	BISC 333 BPK 310 CMPT 120 CMPT 135 MATH 150 MATH 232 MATH 448	BISC 420 BPK 417 CHEM 126 MATH 308	MBB 222 PHYS 485
April 15	ACMA 320 BISC 300 BPK 304 CHEM 363 CHEM 459 EASC 205 EASC 403	BPK 303 BPK 311 PHYS 344	BISC 337 BISC 472 MACM 316 MATH 157 MATH 240 MATH 314	CMPT 135 CMPT 225
April 16	BISC 302 BISC 366 CMPT 371 MATH 242 MATH 251 PHYS 321 STAT 475	CMPT 165	BPK 201 MATH 158 PHYS 415	MATH 252 MATH 303 MATH 320
April 17	BPK 305 CMPT 404 HSCI 100 HSCI 442 STAT 100	CMPT 276 PHYS 121	BPK 342 MACM 101 MBB 322 STAT 101	ACMA 340 BPK 325 EASC 208

	Start Times			
Date	8:30	12:00	15:30	19:00
April 18	CHEM 316	BPK 143	BISC 422	MATH 441
	EASC 400	BPK 407	EASC 307	STAT 270
	PHYS 390	CHEM 260	STAT 240	
April 19	BISC 101	BISC 102	BISC 405	CMPT 213
	CHEM 372	BISC 313	BISC 405	CMPT 310
	CMPT 475	BPK 446	BPK 307	
	EASC 101	CMPT 383	BPK 340	
	HSCI 140	HSCI 120	CMPT 295	
	HSCI 212	MATH 396	MBB 231	
	MATH 208W	MBB 331	PHYS 445	
	STAT 410	PHYS 365		
		STAT 403		
April 20	BPK 207	CHEM 317	CHEM 332	ACMA 470
	CMPT 300	CHEM 460	CMPT 307	BISC 204
	MACM 201	CMPT 129	SCI 304	BISC 309
	PHYS 326		MATH 190	EASC 311
			MATH 341	
			MBB 201	
			STAT 445	
April 21	BPK 105	BPK 241	HSCI 432	BPK 448
	CHEM 122	CHEM 111	MATH 155	HSCI 333
	EASC 204	MBB 438	MBB 342	STAT 341
	MATH 380W			
	PHYS 190			
April 22	BPK 421	BISC 318	PHYS 101	
	CMPT 354	BPK 140	STAT 285	CHEM 281
	CMPT 376W	CHEM 391		HSCI 330
	HSCI 305	HSCI 340		
	MATH 100	MBB 321		MATH 152
				MBB 323
April 23	EASC 210	BPK 343	BPK 141	CHEM 121
	EASC 305	CHEM 336	PHYS 126	
	MATH 462W	MACM 203		
	PHYS 141	PHYS 285		
		PHYS 465		

7 Analysis

The aim of this non-smooth problem (NSP) was to maximize distances between exams and reduce conflicts in scheduling. This problem was solved using an Analytic Solver Platform and the Standard Evolutionary Engine. To summarize our results, the following table shows the total number of exams scheduled for each time slot for each exam date.

	Start Times			
	8:30	12:00	15:30	19:00
April 12	2	7	2	3
April 13	4	6	6	5
April 14	6	7	4	2
April 15	7	3	6	2
April 16	7	1	3	3
April 17	5	2	4	3
April 18	3	3	3	3
April 19	8	9	6	2
April 20	4	3	8	4
April 21	5	3	3	3
April 22	6	5	2	4
April 23	4	5	2	1

Of the exams scheduled at the same time, there are 41 exam conflicts, out of the 2019 course combinations taken by students this semester. Therefore, according to the results from this problem, only 2.0% of the combinations experience conflicts. This further shows that 110 students out of the 7828 total students in the faculty of science (undergraduate) experience conflicting schedules, which is 1.4%.

7.1 8 Limitations and Improvements

Due to the simplification of the problem, this model schedules exams under a few assumptions. First, we made the assumption that there is only one SFU campus. The addition of multiple campuses into this model would have increased the complexity significantly, as different schedules should have to be generated for the different campuses and the travel time between difference campuses should also be taken into account. This model can be further developed to take the different campuses into account. In doing so, we would be incorporating the different class sections for each course (for example D100, E100), and not only track which course combination students take per semester, but also which class sections they are part of, and which campus each section is taught in.

This model can be improved further by adopting a proven or commonly used penalty system. As mentioned above, the penalties were scaled to best represent the relative weight of the conflicts in scheduling. Due to time constraints, the problem was not tested for different penalty values. Since penalty values highly impact the objective function and therefore the results, the model can be assigned penalty values based on observation of various trials with different penalty values. Whichever set of penalty values result in lesser conflicts and longer breaks between course combinations, can be concluded as those best representing our needs.

Since an evolutionary solver was used to solve this problem, the results are dependent on the starting values. Although this model was run a few times and the maximum objective function value was used as the solution, it was not tested with many different starting points. Increasing the number of trials with different starting values could've potentially lead to a better solution.

8 References

Simon Fraser University Engaging the World. (n.d.). Retrieved April 09, 2018, from https://www.sfu.ca/students/exams/spring-term.html

Analytic Solver for Excel. (2018, January 03). Retrieved April 09, 2018, from https://www.solver.com/analytic-solver-platform