Airline Schedule Planning: Optimization Approaches *

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Abstract

This paper discusses the scheduling of Air Canada's planes across a network of cities. Where and when to fly planes are decisions faced by airlines worldwide. The two models used here were able to produce a daily schedule for seven Air Canada planes flying between seven cities, that optimizes profit levels. The obtained schedules are found to be very reasonable.

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1 Introduction

Plane scheduling is a problem that has existed for as long as airlines have. The process of obtaining a cost effective schedule for planes has a high level of complexity as individual planes work job scheduling models within a network model containing all planes. The desire to formulate a model on this problem stems from the financial troubles that Air Canada had in the mid 2000's. Starting with this motivation, and narrowing down the topic, we chose to formulate two models that can create a viable daily schedule for Air Canada.

Air Canada was founded in 1963 and is currently Canada's largest airline, ranked in the top 20 worldwide. It has a fleet of 196 airplanes of varying types, and averages 1,530 scheduled daily flights to various locations all over the world. Although Air Canada is no longer in the aforementioned financial trouble, the models' aim is to make an effective schedule that gives Air Canada the maximum possible net income, thereby giving them the best possible chance at avoiding future debt. The models are designed purely to maximize profit, although to some degree customer satisfaction is addressed indirectly. The more consumer demand is met, the more profits are made. Additionally, some efforts can be made to reduce flights during significantly early morning hours.

The parameters and data used to solve the problem come from the Air Canada Website, which is referenced at the end of the report along with other sources. Since the models in this report were significantly simplified, the data had to be modified slightly and may not give the most realistic portrayal of Air Canada's situation. As well, much of their information is not released and had to be estimated.

Within this paper, the two created models will be introduced, along with their respective assumptions, methods, variables, objective functions and constraints.

2 Background

The problem was first simplified in order to approach it in an effective manner. In reality, Air Canada flies to 175 airports across 5 continents. However, we reduced our plane scheduling to take into account 7 major Canadian cities: Vancouver, Edmonton, Calgary, Winnipeg, Toronto, Ottawa and Montreal. We also focussed purely on the Boeing 787 plane. This was our plane of choice due to its mid size capacity, top class fuel efficiency and its ability to be effective in use for both long and short distance journeys. We considered the scheduling of 7 of these planes.

3 General Assumptions

To simplify our model further we needed to have some overarching assumptions, these assumptions are:

- No Costs associated with being on the ground
- Maintenance costs are based on length of flight
- The length of each flight is rounded to the nearest hour
- Each airport has equal turnover times
- Equal prices for each seat on a plane - no seating divisions (first class, economy, etc.)

4 Model 1

- **Assumptions:**
  - No demand for connecting flights
  - Planes will only fly when full due to excess demand
- **Decision Variables:**

  The only decision relevant to this simplified model is how many times the arc i,j is travelled by a plane p. This is the reasoning behind leaving this variable as a non-negative integer rather than finding a way to include binary conditions.

  $X_{i,j,p}$ = number of times arc i, j is travelled by plane p
  $X_{i,j}$ = number of times arc i, j is travelled by all planes

- **Parameters:**

  $D_{i,j}$ = demand for flight from city i to j
  $P_{i,j}$ = profit of a flight from city i to j
  $t_{i,j}$ = time to travel from city i to j

- **Objective Function:**

  The objective function is designed to maximize profit over all of the planes for a 24 hour period. It also considers the demand served and the amount of flights taken. Because these other factors were much smaller than the amount of profit, weighting was introduced such that all factors were approximately equivalent. With this weighting the demand and amount of flights effectively cancel out, leaving only profit in the objective function.

  $Z = \sum_{i=1}^{7} \sum_{i=1}^{7} (P_{i,j} * X_{i,j} - 450S_{i,j} + 100000X_{i,j})$
  Where $S_{i,j} = 250 * X_{i,j}$

- **Constraints:**

  Because every plane is assumed to fly full at 250 passengers, the amount of passengers per plane times the number of flights travelling an arc must be less than or equal to the total amount of people wanting to travel that arc:

  $$250 * \sum_{p=1}^{7} X_{i,j,p} \leq D_{i,j}$$

  In our model, each airport has a flow of planes in and out. Not only does this constraint ensure that if a plane enters an airport it also leaves, the constraint also forces the plane to return to its starting location at the end of each day. This makes the schedule easily repeatable:

  $\sum_{i=1}^{7} X_{i,j,p} = \sum_{i=1}^{7} X_{j,i,p}$ for each node j and plane p

  This constraint defines the decision variable as a non-negative integer:

  $$X_{i,j,p} \in 0, 1, 2, 3, ...$$

  The final constraint keeps track of how long a plane has been flying for the period and makes sure that it is within the total time allotment of 24 hours:

  $$\sum_{i=1}^{7} \sum_{j=1}^{7} t_{ij} X_{ij} \leq 24$$

5 Model 2

- **Assumptions:**

  Here the demand is a soft constraint within the objective function. The number of people wanting to travel from one city to another is an entered parameter that does not change according to prices, (the scope of this model does not include market analysis and elasticity). In order to facilitate the scheduling of connecting flights, it is assumed that if the price of a direct flight between two cities was greater than the price to two flights (from the first city to an intermediate city, and from the intermediate city to the final destination), then one tenth of the people would choose the indirect flight connection. This model does not take into account the fluctuations of prices or demand for flights at different times of day.

- **Decision Variables:**

  X_{ijpt} = 1 if plane p flies from city i to j starting at time t, and 0 otherwise.

  Y_{ij} = an accessory integer variable, specific to each arc i, j.

- **Parameters:**

  S_{ip} = 1 if plane p starts the day in city i, and 0 otherwise.

  L_{ij} = the length of a flight from city i to j, rounded to the nearest hour and including a 1 hour stopover.

  G = the number of seats on a plane.

  C_{ij} = the cost of one flight from city i to j.

  D_{ij} = the number of people wanting to go from city i to j.

  P_{ij} = the price of a ticket for a flight from city i to j.

  U_{ikj}= 1 if it is cheaper to take a connecting flight from between city i and j stopping over at city k, and 0 otherwise.

  $U_{ikj} = \ulcorner(sin(\frac{P_{ij}-P_{ik}-P_{kj}}{BigM}) + 1)\urcorner - 1$

  B_{ij} =the effective demand for seats on planes from city i to j.

  $B_{ij} = D_{ij} - \sum_{k=1}^{7}(\frac{1}{12}U_{ikj}D_{ij}) + \sum_{k=1}^{7}(\frac{1}{12}U_{ijk}D_{ik}) + \sum_{k=1}^{7}(\frac{1}{12}U_{kij}D_{kj})$

- **Building the Objective Function:**

  V_{ij} = the total number of planes that fly from city i to j within the time period.

  $V_{ij} = \sum_{p=1}^{7}\sum_{t=1}^{2} 4X_{ijpt}$ (for each i,j)

  $Y_{ij} \leq B_{ij}P_{ij}$

$Y_{ij} \leq V_{ij}P_{ij}G$

$W_{ij} =$ the revenue from the total flights on a specific arc.

$W_{ij} = Y_{ij}$

$Q_{ij} =$ the total cost of the flights ran on a specific arc.

$Q_{ij} = C_{ij}V_{ij}$

$Z =$Profit

$Z = \sum_{j=1}^{7}\sum_{i=1}^{7}(W_{ij} - Q_{ij})$

Z is maximized.

- **Constraints:**

  No plane can take off in two locations at the same time:

  $\sum_{j=1}^{7}\sum_{i=1}^{7} X_{ijpt} \leq 1$ for each p, t

  If a plane takes off heading towards an airport, then it must depart from that airport after the length of time it takes to fly there, except for the last flight of the day:

  $X_{ijpt} - \sum_{k=1}^{7} X_{jkp(}t + L_{ij}) \leq 0$ for each i, j, p, t; t$\leq$ $(24 - L_{ij})$

  If a plane takes off from an airport, then it cannot take off from another airport for the duration of its flight and stopover time:

  $(1 - X_{ijpt}) - \sum_{j=1}^{7}\sum_{i=1}^{7}(\sum_{n=1}^{L_{ij}-1}(X_{ijp(t+n)})) \geq 0$ for each p,t; $t \leq (24 - L_{ij})$

  A plane cannot take off on a flight that will not land before the end of the time period in consideration (24 hours):

  $\sum_{j=1}^{7}\sum_{i=1}^{7}(\sum_{n=0}^{L_{ij}-2} X_{ijp(24-n)}) = 0$ for each p

Each plane must leave from its designated start node at the beginning of the day:

$$S_{ip} - \sum_{j=1}^{7} X_{ijp1} = 0 \text{ for each i, p}$$

6 Study Results

Model 1 consisted of 343 variables and 448 constraints. Using OpenSolver software, an optimal solution was found almost instantaneously providing us with a working daily schedule for Air Canada consisting of purely direct flights. Using the estimated demands based off of Air Canada's data our model resulted in a daily gross income of $6,051,891. This income was attained by serving 13,250 customers.

This model is very flexible and we were able to investigate possible uses to improve emergency damage control and customer service.

One of these emergency scenarios is the closing of airports. To model this we completely closed airports as they would in the case of an emergency and took note of the loss of revenue. When closing Montreal, the busiest Airport by our model, we lost only $880,164. Also our model was able to adapt and route different flights in such a way two less traverses of a flight arc were able to be run. This result demonstrates that although the monetary loss is significant, the model is able to adjust and still make use of all available plane to earn significant profit.

When taking into account customer service we were able to limit Red-Eye Flights (Flights taking off from 12am-6am). We did this by scheduling the longer flights earlier, this caused less departures in the Red-Eye zone. We were able to limit red eye flights to a total of 12 throughout the entire schedule, while not losing any revenue as the flights didn't change, just the order in which they took place.

Model 2 consists of 7,098 variables and 13,734 constraints. Using OpenSolver software, an optimal solution was found after approximately 20 minutes. This solution provided us with a working daily schedule for Air Canada that was able to take into account both direct and connecting flights. It resulted in a daily gross income of $5,859,914, and accomplished this by serving 11,250 customers. Because the schedule takes into account the starting locations of each plane, the results can vary depending on where they start the day. Different starting scenarios were considered, and of those outcomes achieved the profit was greatest when all of the planes started at different airports. The finishing airports for each plane can be re-entered as the starting locations, and multiple days can be reasonably scheduled this way.

7 Discussion Of Results

Although these gross income values are very high for a single day, it must be noted that within our model took very little overhead costs into account. The only costs taken into account were fuel and a very broad estimate of maintenance costs that was based on length of flight. More detailed overhead costs were not taken into account as the data cannot be found on-line as it isn't readily released to the public.

In both models, the flights between Montreal and Toronto and vice versa were occurred the most frequently. This is likely due to their relatively high ticket price, and low costs (as it is a fairly short distance). Because the number of planes scheduled was quite small, many of the arcs were not actually flown, most probably due to their relatively high cost to revenue ratio. An interesting variation on the models could consider enforcing the frequency of arc usage to be more evenly distributed.

Although between our two models we can create very flexible schedules offering different flight plans, further improvements can still be made. One large area for possible improvement is to increase the number of planes and airports the models can handle. With just 7 airports and 7 planes the usefulness of the models is limited.

Another area that could be improved upon is the flexibility of the second model. By flexibility we are referring to the ability to run various scenarios. However, there is cause for concern when attempting to improve its flexibility as any changes would likely result in an increase in complexity. This increased complexity could cause the time needed to solve for a feasible schedule to increase greatly, rendering the model less useful.

These adjustments would potentially make the models more effective on a larger scale and allow them to be more responsive to their external environment.[†]

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