A Queueing Network Model for Refugee Language Courses in Vancouver

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Abstract

A surge of over 1,600 landed refugees in Vancouver has caused a spike in demand for English language courses resulting in longer than usual wait lists. With no accurate information regarding expected wait times, refugees often choose suboptimal waiting queues, contributing to the large variance observed in wait times. The purpose of this paper is to explore the benefits of a queueing network which assigns refugees to queues based on the expected wait times, with the intent of reducing total service times given the current resources. Furthermore, a range of priority policies are explored in order to observe their effect on the increasing refugee student population. By considering the distinct wait lists that form for each level of language course within each learning centre, a network of queues can be devised with each queue represented by a node. The nodes are organized by course level l, with each node in level l connected to each node in level $l+1$ by a unique directed edge. For each queue in the network, the expected wait and service times are approximated using factors such as class capacity, current queue length, and departure rate. This queueing network solution reduces expected wait times, lowers the variance in wait times, and results in faster integration of refugees into Canadian communities and the labour force.

1 Introduction

In the past year, there has been a significant increase in the number of refugees arriving to the City of Vancouver. With this increase comes a spike in demand for Language Instruction for Newcomers to Canada (LINC) courses, a government funded language program for refugees and landed immigrants. Prior to the current influx, refugees had reported extreme wait times of up to 12 months [17], signaling a looming problem that may worsen if nothing is done. On the other hand, others have reported waiting under a month for entry into courses, raising the question of balance and fairness in the distribution of people across centres and wait lists [14].

The imbalance in wait lists stem from a series of misinformed choices made by newcomers to the LINC system. The current system allows for a newcomer to join multiple LINC centres, which inflates the wait lists and misrepresents the expected wait times. Newcomers who attempt to choose a LINC centre based on estimated wait times may find themselves choosing a suboptimal queue. Choices made arbitrarily or out of preference and convenience have a chance of creating a balanced spread across centres, however it is not guaranteed. Finally, the current influx of refugees may hinder the wait times of others in the system, requiring a reconsideration of course acceptance priorities to stabilize the flow in the long term.

The purpose of this paper is to address the above issues in a manner as to optimize the flow of refugees through the LINC course system. Firstly, a new model is proposed that can accurately estimate the expected wait times for any course by restricting students to enroll in only one course. Secondly, deterministic *assignment policies* are proposed to optimize and balance the flow of the system as a whole. Lastly, different *priority policies* are looked at to find a policy that increases the rate at which refugees pass through courses without significantly hindering other groups of students.

The model used is that of a queueing network that describes the flow of newcomers through the LINC course system. With course offerings represented by queue nodes, the nodes are organized into course levels with members transitioning to a course of the next level. The flow of the network is dictated by the assignment policy, and the members entering the course from the queue are chosen as per the priority policy.

The surge in refugees motivates a response in the form of two policy solutions. A *stable policy* solution is an assignment policy and priority policy that optimizes the flow of the network during normal flow. A *surge policy* solution is an assignment policy and priority policy that optimizes the flow of the network while prioritizing a select group without significantly hindering others in the system. This policy is designed to take effect when a surge is observed, and stay in place until the flow returns to stability.

A simulation of the network is tested for various policies in Section 6 and 7, and the benefits of a deterministic assignment policy are observed. Furthermore, the effects of various priority policies on the network flow are explored during both stable and surge conditions.

1.1 Background

1.1.1 The LINC Program

As a means of assimilating refugees and immigrants to Canadian culture, the government provides language courses known as LINC to eligible permanent residents free of cost. LINC classes are designed to improve the language skills and boost the employability of newcomers to Canada. Attaining a certain language skill level is also a requirement for some refugees to attain Canadian Citizenship, making the LINC program essential for many refugees.

1.1.2 Syrian Refugees in Vancouver

In 2014, 1,290 Syrian refugees migrated to Canada and accounted for only 5.5% of the 23,285 arriving refugees that year [15]. As part of the Canadian government's initiative to bring in more Syrian refugees, 26,202 Syrian refugees have arrived in Canada [2] with over 1,600 arriving in the City of Vancouver between November 4th, 2015 and March 20th, 2016.

Due to the large increase in refugee arrivals and their need for LINC courses, there has been a notable increase in demand and wait times for LINC courses in Vancouver, which has received much media coverage. Many articles report some refugees are waiting long periods of time to get into classes while other are able to get into classes almost immediately [14]. These reports provide evidence of the large variation in wait times for courses and indicate that some refugees are spending more time than necessary in waiting lists.

2 Scope of Study

This study considers 122 LINC courses across 7 LINC centres in Vancouver. The details and definitions surrounding this choice of scope follow.

2.1 LINC

In regards to language courses, only government funded LINC courses will be considered in this paper. LINC courses adhere to a standardized curriculum guide as defined by Citizenship and Immigration Canada (CIC) [6], and can guarantee a similar level of quality across all centres.

The scope is further restricted to LINC classes that employ a continuous intake model for refugees. These classes allow for students to begin lessons anytime a seat is available, or advance to a higher level once a student meets the requirements set forth by CIC. This restriction allows for a class to be represented by a queue with probabilistic arrival and departure times. Given that 90% [4] of LINC classes use this convention, excluding classes with fixed semesters will still allow the model to effectively capture the true nature of the system.

Our presented study acknowledges the *immigration classes* of LINC students as defined by the study by Dempsey, Xue, and Kustec (2009) [7]. This study categorizes students into Refugees, Skilled Workers, Family Class, and Others. Including the different categories in the scope of this paper establishes a more dynamic model capable of determining the optimal policies given a surge in members of any immigration class.

2.2 Geographical Scope

With the city of Vancouver as the geographical focus of this paper 1, LINC centres in neighbouring cities such as Burnaby and Richmond are not included in the Queueing Network Model. Thanks to Vancouver's numerous transit options and route availabilities, LINC students can travel between all centres within this geographical region with negligible cost and minimal time. This means that a student can transfer from one centre to another upon completion of a course if it is condusive to reducing their wait time as it is consistent with our assumption of unbiasedness of centre location (Section 3.1.9).

3 Model

The model used to describe the LINC system is that of a directed queueing network. Within the network, each LINC class is represented by a *queue node* while a newcomer joining the system is represented by an *arrival node*. These nodes are organized within the network in stages, or *levels*, with directed edges connecting a node with level l to a queue node of level $l+1$. This represents the flow of LINC students entering the system or graduating from a course, and applying for the next available course level.

The arrival rate into the network is assumed to follow a Poisson Process, where the inter-arrival times are exponentially distributed with arrival rate λ. By the data collected for departure time means and with sufficiently large sample size [18], by the Central Limit Theorem [8] the departure times are assumed to follow a Normal distribution, with mean departure time $\frac{1}{\mu}$. Since each queue node has an exponential arrival and normal departure distribution, and has a class capacity

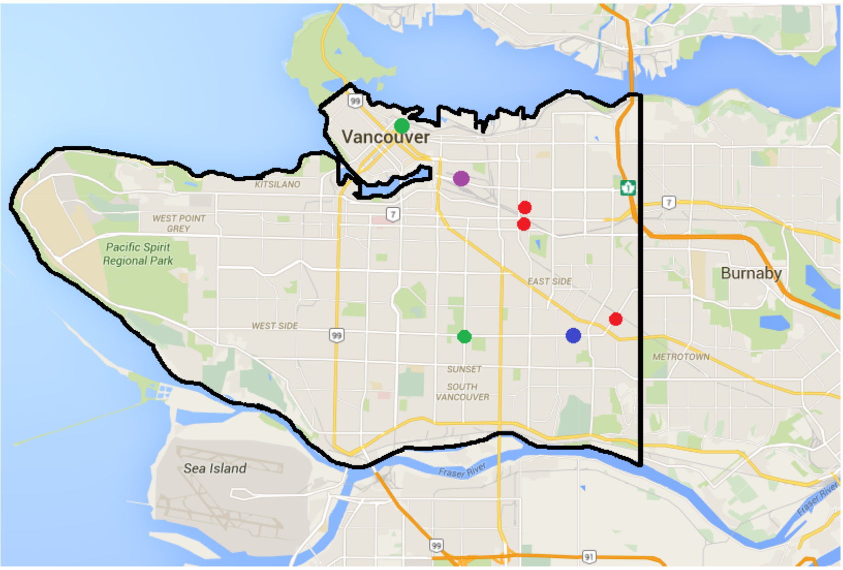

Figure 1: A map of the LINC centres in the City of Vancouver [9]

greater than 1, each queue node can be seen as a $M/G/c$ queueing system [10].

The flow of LINC students through the queueing network is dictated by the *assignment policy* and the *priority policy*. The assignment policy determines the queue node a LINC student will transition to next, while the priority policy determines the next student in the queue selected to enter a class. A *policy solution* of the model is a pair of an assignment policy and a priority policy that optimizes the flow of the network.

With respect to the increasing number of refugees entering the LINC system, it is of interest to consider the effects of different assignment policies and priority policies on the departure time of refugees and on the system as a whole. For this, two policy solutions are considered: a *stable policy* and a *surge policy*. The stable policy is a policy solution intended to optimize the LINC system during normal flow conditions. The surge policy is a policy solution that takes effect during a surge of a particular group, such as that of refugees in Vancouver, and is intended to prioritize the flow of that group without significantly hindering the flow of others.

3.1 Network Construction

The network is defined as a directed graph $G = (N, E)$ where N is the set of nodes and E is the set of directed edges. M is the set of members currently in the network.

The network holds the following properties:

1. A node $i \in N$ can be a queue node $q \in Q$ or an arrival node $a \in A$. Therefore, $N = Q \cup A$.
2. A queue node $q \in Q$ has the following attributes:
    - c: The member capacity of the course.
    - s: The number of seats filled, $s \leq c$.
    - f: The frequency of the course, in hours per week.
    - $\mathcal{W}$: The set of members in the current wait list for the course.
3. The levels of the network are $l \in \{0, 1, ..., L\}$.
4. Queue nodes $q \in Q$ are present in the last L levels such that $Q = \{\mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_L\}$, $\mathbf{q}_l = \{q \in Q | \mathcal{L}(q) = l\}$.
5. There exists exactly one arrival node for the first L levels such that $A = \{a_0, a_1, ..., a_{L-1}\}$.
6. The level of a node $i \in N$ is described by the *level function* $\mathcal{L} : N \rightarrow \{0, 1, ..., L\}$, $\mathcal{L}(i) = l$.
7. For any pair of nodes $i \in N$ and $q \in Q$, there exists a directed edge $(i, q) \in E$ if and only if $\mathcal{L}(i) + 1 = \mathcal{L}(q)$.
8. For a node $i \in N$, the set of all queue nodes $q \in Q$ that share a directed edge $(i, q) \in E$ with i is denoted by $Q(i)$.
9. Each directed edge $(i, q) \in E$ has two costs. The flow of the network may depend on one or neither of these costs.
    - The *waiting cost* is $w_{iq}(m)$, where $m \in M$ is a member travelling from i to q. (Figure 3)
    - The *total service cost* is $t_{iq}(m)$, where $m \in M$ is a member travelling from i to q. (Figure 3)
    - Travel cost between centres is negligible assuming unbiasedness of centre location.

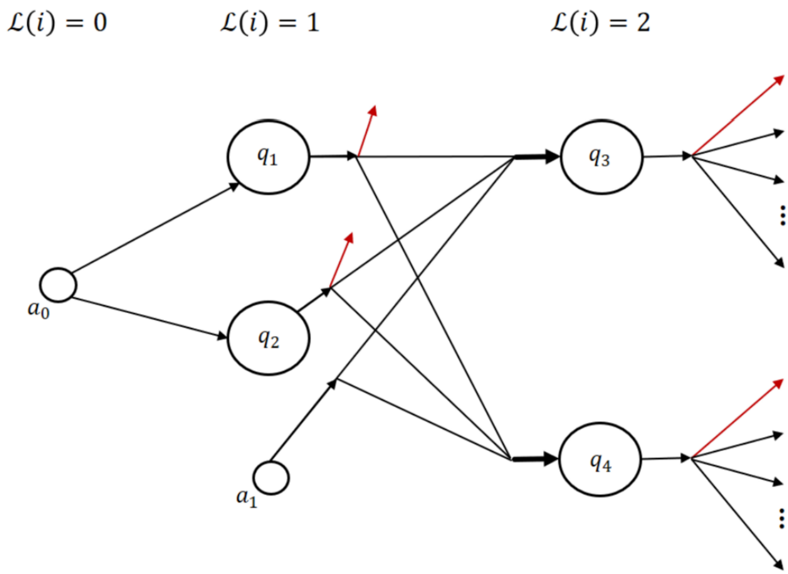

Figure 2: The basic construction of the Queue Network

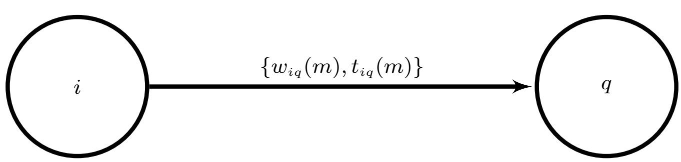

Figure 3: A directed edge $(i, q) \in E$ has a wait cost $w_{iq}(m)$ and a total service cost $t_{iq}(m)$

3.2 Arrival of Newcomers

3.2.1 Source

Prior to entering the LINC system, newcomers must go through an assessment process, which determines the language skill level of the newcomer. These assessments are of similar length throughout the day, creating a steady stream of newcomers exiting the assessment process. A research article by Whitt(1982) [19] shows how the Iglehart-Borovkov Limit Theorem [11] can be used to simplify a system with exponential service times to have a Markovian service rate when the system is under heavy traffic. Assuming an exponential service time at the assessment center, the data collected in section 4.3 shows the assessment centre is functioning at capacity, and therefore qualifies as being under heavy traffic such that the Iglehart-Borovkov limit theorem applies. Since the assessment process feeds directly into the LINC system, the arrival process of the queueing network can also be assumed to be Markovian.

3.2.2 Arriving to the Network

The arrival rate to the system is denoted by λ, where the inter-arrival time is exponentially distributed. For any arriving member m, the assessed skill level of the member is l with probability p_l. Therefore, the arrival rate of a member at level l to the system is $\lambda_l = \lambda p_l$. These assessment probabilities p_l are assumed to remain constant over short periods of time of under 5 years.

For newcomers assessed at higher levels, they may decide that their language skills are strong enough to not require any courses. For each level, the probability that a member m assessed at level l will choose not to pursue courses is the *discontinuing probability* and is denoted by ρ_l. This probability is monotonically increasing with respect to l such that $\rho_\alpha \leq \rho_\beta$ when $\alpha < \beta$ for $\alpha, \beta \in \{0, 1, ..., L\}$. The probability that this member will choose to pursue courses is $1 - \rho_l$.

Altogether, the arrival rate of a member to the network at a_l is $\lambda_l^* = \lambda p_l(1 - \rho_l)$.

3.2.3 Arrival Rate for Queue Nodes

In the presence of a probabilistic assignment policy and a First-In-First-Out (FIFO) priority policy, the arrival rate to each queue node can be solved analytically. However, deterministic assignment policies and non-FIFO priority policies will be considered, making it impractical to solve for the arrival rates. Instead, the arrival rates will be looked at and compared in Section 5 when the results of the simulation are analyzed.

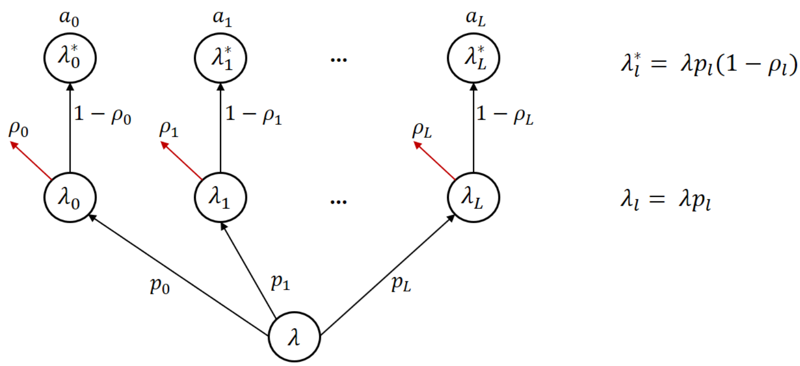

Figure 4: The arrival process of members to the system.

3.3 Departure Rate

In place of a set of servers at the end of the queue, each member in a LINC class can be viewed as their own server with mean departure time $\frac{1}{\mu}$ hours following an exponential distribution. As with the assessment processes, the Iglehart-Borovkov limit theorem [11] can be applied since the majority of the LINC system is at capacity and qualifies as being under heavy traffic, allowing the departure rate of each queue node to be Markovian. Therefore, for a queue node $q \in Q$ with capacity c and frequency f, the departure rate of q is $cf\mu$ in weeks.

3.3.1 Simplifying the Departure Rate

From the collected data in section 4.3 it was observed that for each immigration class $c \in \mathscr{C}$, there was a different mean departure rate $\frac{1}{\mu_c}$. Although at any given time the departure rate of a queue node $q \in Q$ with capacity c and frequency f could be calculated as

$$cf \sum_{m \in q} \mu_m$$

, the calculations become increasingly challenging when estimating the expected wait times.

Consider a queue node $q \in Q$ at capacity with K different immigration classes, each with its own departure rate μ_k. For a capacity c and frequency f, the expected time until the next departure is

$$\frac{1}{f \sum_{k=1}^{K} c_k \mu_k}$$

where c_k is the number of members of immigration class k. If a member arrives in the queue and is second in line, the expected wait is

$$\frac{1}{f\sum_{k=1}^{K} c_k\mu_k} + E(q_A)$$

where $E(q_A)$ is the expected wait time once one member has left the class and the next member in the queue has joined. $E(q_A)$ can be calculated by taking the expected wait of each potential system times the probability of that system occuring. Then,

$$E(q_A) = \sum_{\alpha \in A} \left(\frac{\mu_\alpha}{\sum_{k=1}^{K} c_k\mu_k} \right) E(q_\alpha)$$

Note that $E(q_A)$ required the sum of c expected waits. If a member arrives and is third in the queue, the expected wait is

$$\frac{1}{f\sum_{k=1}^{K} c_k\mu_k} + E(q_A) + E(q_B)$$

where $E(q_B)$ is the proportional sum of all combinations of any 2 members leaving, which requires the sum of $\binom{c}{2}$ expected waits.

Although it is possible to estimate the expected wait with higher degrees of accuracy using the above method, the absence of a closed form calculation make it inefficient to use for our simulations. Furthermore, we can reasonably assume that the departure rate is similar across all members in the long run, hence we assume the departure rate is identical across all members and attain a closed form estimation for expected wait time as a result.

3.3.2 Departure Rate Assumptions

In order to keep the departure rate constant over all members and courses, several assumptions need to be made with respect to the quality of education. Firstly, it is assumed all LINC centres have similar quality, and thus the expected departure time for a member should be equal between two courses of the same capacity and frequency 2.1. Secondly, it is assumed that any outside help or studying, which includes resources such as online lessons and tutorial videos outside of class hours, does not significantly reduce the expected departure time for a member. Lastly, all departure rates are assumed to remain constant over short periods of time of under 5 years.

3.4 Network Costs

As will be seen in section 3.5 when discussing the assignment policies, it's crucial for the network flow to know the expected wait time and expected total service time for any transition. An edge $(i, q) \in E$ has two cost attributes: $w_{i_q}(m)$ and $t_{i_q}(m)$ representing the cost of waiting in queue and the total service cost respectively. Note that the costs depend on m; this is because different priority policies may place m in a different position in the queue.

Consider a member $m \in M$ exiting a node $i \in N$. If $\mathcal{L}(i) < L$, there exists an edge $(i, q) \in E$ connecting node i to queue node $q \in Q$ with attributes c, s, f, and $\mathcal{W}$ (Section 3.1). The expected wait time and expected total service time are defined as follows:

$$w_{i_q}(m) = \begin{cases} 0 & \text{if } s < c \\ \frac{(|\mathcal{W}|+1)}{cf\mu} & \text{otherwise} \end{cases} \tag{1}$$

$$t_{i_q}(m) = \begin{cases} \frac{1}{f\mu} & \text{if } s < c \\ \frac{1}{f\mu} + \frac{(|\mathcal{W}|+1)}{cf\mu} & \text{otherwise} \end{cases} \tag{2}$$

3.5 Assignment Policy

For a member $m \in M$ leaving a node $i \in N$, the choice of the next queue node $q \in Q$ visited is determined by the *assignment policy*. The assignment policy may be *probabilistic* or *deterministic* in nature, and greatly affects the flow of the network.

A probabilistic assignment policy is one such that a member $m \in M$ leaving a node $i \in N$ chooses to join queue node $q \in Q(i)$ with probability p_{i_q}, where

$$\sum_{q \in Q(i)} p_{i_q} = 1$$

This form of assignment may model the real-world situation more accurately, as newcomers seem to choose a centre based on personal preference or convenience. In the absence of notions of preference or convenience, the choices made become arbitrary and converge in the long run to a discrete uniform distribution where $p_{i_q} = \frac{1}{|Q(i)|}$. For simplicity, the proposed model only considers a uniform assignment policy in order to model probabilistic choice behaviour.

A deterministic assignment policy is one such that a member $m \in M$ leaving a node $i \in N$ chooses to join queue node $q \in Q(i)$ based on a set of criteria. Examples of such policies follow:

- *Minimum Expected Wait*. For a member $m \in M$ leaving a node $i \in N$, the chosen queue node q is defined as

  $$q = \operatorname*{arg\,min}_{q \in Q(i)} (w_{i_q}(m))$$

  where $w_{i_q}(m)$ is the expected wait function (Section 3.4, Eq 1).

- *Minimum Expected Service*. For a member $m \in M$ leaving a node $i \in N$, the chosen queue node q is defined as
$$q = \underset{q \in Q(i)}{\arg\min}(t_{iq}(m))$$
where $t_{iq}(m)$ is the expected total service function (Section 3.4, Eq 2).

- *Shortest Queue*. For a member $m \in M$ leaving a node $i \in N$, the chosen node q is defined as
$$q = \underset{q \in Q(i)}{\arg\min}(w)$$
where $w = |\mathcal{W}|$ is the wait list length of q from property 2.

Although the Shortest Queue policy may seem like a strong indicator of the expected wait time, the differences in class sizes and frequencies have a strong impact on the actual wait time. As a result, a short queue for a small class or a class that does not meet often will not result in a seat as quickly as an average queue for a large or frequent class.

On the other hand, both the Minimum Expected Wait and the Minimum Expected Service policies are calculated using the class sizes and frequencies resulting in a more accurate estimation. These two policies will be used during the simulation analysis in section 6.

3.6 Priority Policy

For a queue node $q \in Q$ with a waiting list $\mathcal{W}$, the next member $m \in \mathcal{W}$ to fill a vacant seat in the course is determined by the priority policy. Each member m belongs to a *priority class* $p \in \mathcal{P}$, with m_p denoting a member m belonging to priority class p and $\mathcal{W}(p)$ denoting the number of members $m \in \mathcal{W}$ with priority class p. Each priority policy describes a criteria for selecting the next member $m \in \mathcal{W}$ with respect to their priority class p. Examples of such policies follow:

- *First-In-First-Out (FIFO)*. The FIFO policy describes the lack of priority. Members in a wait list are processed in the order of their arrival, without considering their priority class.

- *Round-Robin*. For a set of priority classes $\mathcal{P}$, a cycle is defined as a repeating ordered set of priority classes $p \in \mathcal{P}$. When a seat in the class becomes vacant, the next selected member $m \in \mathcal{W}$ will be from the priority class p that is next in this cycle. Within each priority class, members are processed using FIFO.

- *Balanced Wait List*. For a set of priority classes $\mathcal{P}$, let h_p describe the historical proportion of $p \in \mathcal{P}$, $\sum_{p \in \mathcal{P}} h_p = 1$. Let b denote a *balance window* which describes a threshold of significant increase in member volume, $0 \leq b < 1$. For any priority class $p \in \mathcal{P}$, if $\mathcal{W}(p)/\,|\mathcal{W}| > h_p + b$, then members m_p are prioritized.

3.6.1 Studied Priority Policies

For the purpose of this paper, four priority policies will be considered:

1. A *FIFO* policy. This will be used to study the normal flow of the network and to allow for extensive analysis on the assignment policy.

2. A *Round-Robin* policy with *immigration classes* as the priority class. The two immigration classes considered are *Refugees* and *Others*, the latter of which encapsulates all remaining immigration classes.

3. A *Round-Robin* policy with *incoming centres* as the priority class. For a queue node $q \in Q$, the incoming centres are labeled as *Within* and *Outside*, making a distinction between members transitioning to a course after completing the previous course within the same institution, and those arriving to the institution for the first time. *This policy is currently being used by some LINC centres in Vancouver*.

4. A *Balanced Wait List* policy with *immigration classes* as the priority class. The two immigration classes considered are *Refugees* and *Others*, the latter of which encapsulates all remaining immigration classes. This policy in particular will be studied with respect to its behaviour in the presence of a significant increase in a priority class, such as with the increase of Refugees in Vancouver.

3.7 Exiting the Network

For a member $m \in M$, there are a three scenarios in which they will exit the network.

1. *They complete the LINC program*. Since the network has a total of L course levels, once a member attains a language skill level of L, they will have finished the program and exit the network.

2. *They are satisfied with their language skill level*. As the language skill level l of a member m increases, they are more likely to reach their goal language skill level. As seen in section 3.2.2, the probability that a member m assessed at level l chooses to not pursue further studies is ρ_l, which is monotonically increasing with respect to l.

3. *They cannot continue*. Due to extraneous circumstances, some members may not be able to continue. The probability that at any level l a member m cannot continue to the next course is the *dropout probability* and is denoted by a constant χ.

From (2) and (3), it can be seen that for a member $m \in M$ leaving a queue node $q \in Q$ such that $\mathcal{L}(q) = l$, m will exit the network with probability $\chi\rho_l$.

3.8 Assumptions and Simplifications

Many assumptions and simplifications have been discussed up to this point, and are summarized below:

- A member in the network is limited to joining one queue node at a time without the ability to switch (Section 1).
- All courses follow a continuous intake model for new students (Section 2.1).
- The arrival process of each queue node is Markovian (3.2.1) (Section 3.3).
- The departure process of each queue node is normally distributed (3.2.1) (Section 3.3).
- Each queue node can be represented as a $M/G/c$ queueing system (Section 3).
- During normal flow, the arrival rate λ and the departure rate μ remain constant over short periods of time (under 5 years) (Section 3.3.2).
- The assessment proportions p_l remain constant over short periods of time (under 5 years) (Section 3.2.2).
- During normal flow, immigration class proportions remain constant over short periods of time (under 5 years) (Section 4.4).
- The narrow geographical scope renders travel cost between centres negligible (Section 2.2).
- There is no significant difference in quality between two LINC centres (Section 2.1).
- Online lessons and tutorial videos outside of regular class hours do not significantly reduce the expected completion time for a student (Section 3.3.2).

4 Data

4.1 Data Collection Methods

Data regarding LINC courses and centres was collected primarily by email communication with LINC centres in Vancouver [1]. Questionnaires were sent out, with most of the responding centres providing clear answers. The few responding centres that did not provide clear answers, or misinterpreted questions, were sent follow up emails to clarify any ambiguity. Centres that did not respond to the emails were contacted via telephone and asked the identical set of questions as in the email.

Data pertaining to LINC assessment centres was collected from Western ESL Services via a phone interview with the manager [18]. Western ESL Services, who provides language assessment for all newcomers in Metro Vancouver, was able to share the proportions of assessed language skills for newcomers (Table 3). In addition to questionnaires and phone interviews, data was obtained from the study of LINC class profiles (Dempsey et al., 2009) [7].

4.2 Data Composition

For each of the seven considered LINC centre in Vancouver, data on class sizes, wait lists, average completion times, weekly schedule, and monthly attendance rates were organized into tables. All centres provided numerical data on average completion times and weekly schedule, however four centres only provided a range of values for class sizes, waitlists and monthly attendance. This same data offered by the other three LINC centres was considered to be representative of those of the four centres which did not make this information available. The distribution of the class sizes of the three centres which provided this information was used to populate the range of class sizes at the other centres. Similarly, attendance rates and the distribution of the waitlists of these three centres were used to estimate those of the remaining four centres.

4.3 Data Summary

The number of students in a class at any centre varies with time but ranges from 15 to 20 in the majority of cases. The wait list for these classes also change regularly. A notable 25% -30% of classes had an empty queue, and 83-97% of classes had queues with under eleven people in any given month.

According to the LINC profile study (Dempsey et al., 2009) [7], the number of hours it takes a student to complete a class varies according to the studentŠs immigration status, country of origin, and native language. On average refugees take 381 hours to complete a course, nearly 100 more hours than other immigrants who take 283 hours on average (Figure 9). It is worth noting that the average number of hours taken by Arabic speaking immigrants to complete a course is 278 hours; therefore, it is expected that the average number of hours it will take a Syrian refugees to complete a class will range between 278-381 hours. The expected completion time for a class is dependent on the schedule of the class. Morning, Afternoon, Evening, and Weekend classes have different expected completion times as seen in Figure 5.

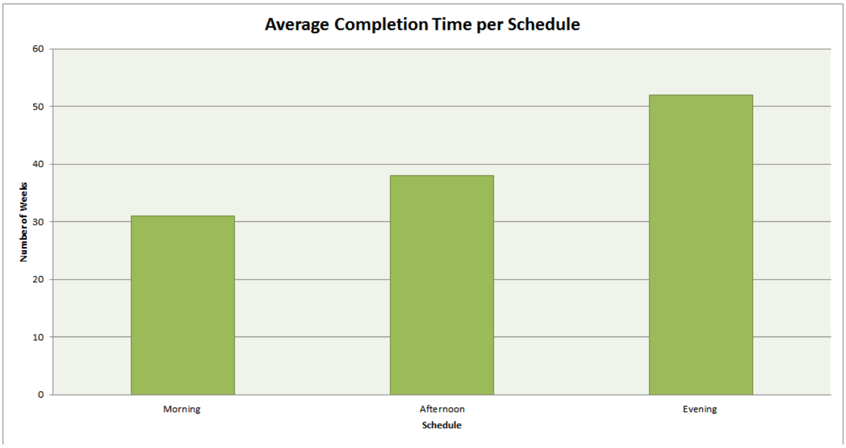

Figure 5: Barplot of Completion time per Schedule

4.4 Defining the Network

Using the collected data (Section 4.1), we describe the data that directly defines the Queueing Network Model.

- The network contains a total of 122 queue nodes over 8 levels. (Figure 6)
- For the arrival rate λ, the proportions p_l of members assessed at level l are described in Table 3.
- The distribution of the frequency values f across our network is described in Figure 7.
- The distribution of the capacity values c across our network is described in Figure 8.
- The proportions im_c of immigration classes is described in Figure 9. These proportions are assumed to remain constant over a short period of time under 5 years.
- The mean departure times of each immigration class is described in Figure 9. For the network, we use a proportional average departure rate such that

$$\mu = \sum_{c \in \mathscr{C}} im_c \mu_c$$

where im_c and μ_c and the proportions and the mean departure times from the figure.

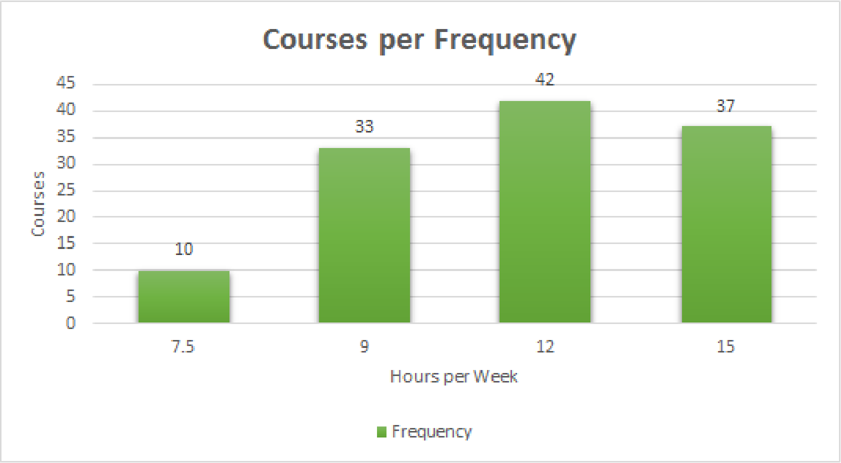

Figure 6: LINC Courses per Level [1]

The arrival rate λ and the exiting probabilities ρ_l and χ are not defined by the collected data, and are instead estimated during calibration of the simulation in Section 5.4.

Table 1: Immigrant Class Proportions

Immigrant Class	Refugee	Family Class	Skilled Worker	Other
Proportion	0.20	0.40	0.30	0.10

Table 2: Immigrant Class Proportions

Immigrant Class	Refugee	Family Class	Skilled Worker	Other
Proportion	0.20	0.40	0.30	0.10

Table 3: Immigrant Class Proportions

Immigrant Class	Refugee	Family Class	Skilled Worker	Other
Proportion	0.20	0.40	0.30	0.10

5 Simulation

Our simulation was programmed using the SimPy package in Python [16], a process-based discrete event simulator. During the simulation, members arrive to the system at an arrival rate

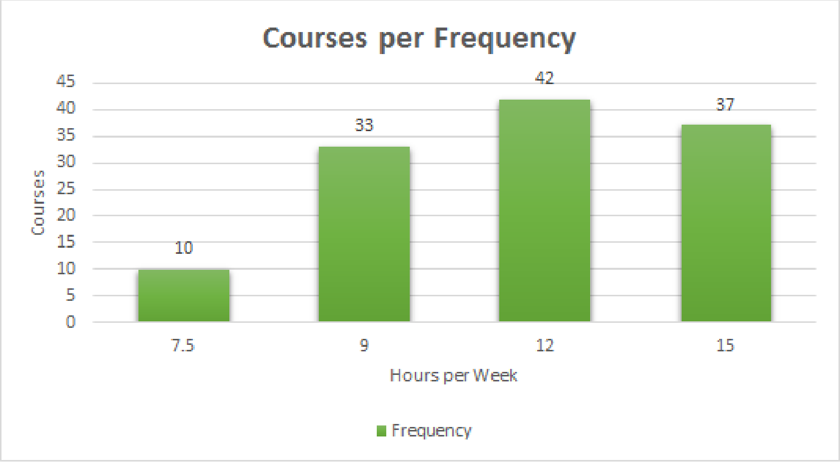

Figure 7: Frequency of LINC Courses [1]

Figure 8: Capacity of LINC Courses [1]

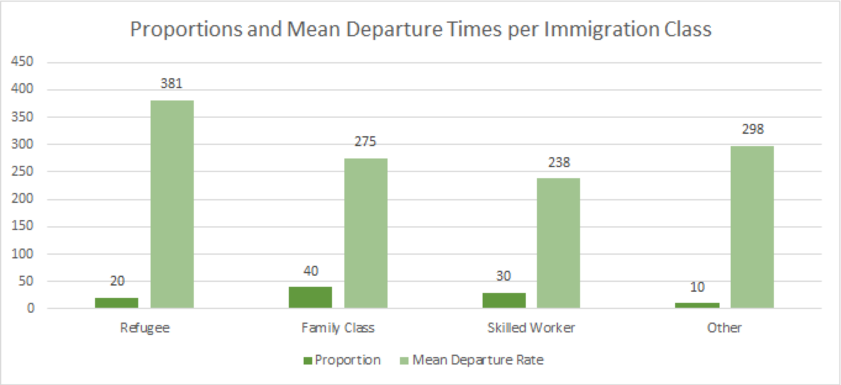

Figure 9: Proportions and Mean Departure Rate per Immigration Class [7]

λ, are assessed at level l with probability p_l and identified as immigration class c with probability im_c. The inter-arrival times of members is exponentially distributed. For a given priority policy, we make use of SimPy's PriorityResource class to simulate the ranking of members in the queue based on our criteria.

In addition to stable network flow, a *surge* is implemented into the script to double the number of refugees in the system for a short period of time. The surge occurs for a total of 16 weeks to mirror the arrival of refugees into Canada from November 2015 to March 2016. Refugees normally arrive at a rate of $\lambda im_{Refugee}$ = 0.2λ, and should therefore represent approximately 20% of all members in the network at any given time during normal flow. When the surge takes place, the overall arrival rate increases by 20%, and the proportion of refugees in the system increases to 30%.

5.1 Discrete Event Simulation

Discrete Event Simulation (DES) involves the simulation of systems where events occur at random and discrete times [13]. With respect to our simulation, events are calculated and placed in an action queue. The simulation jumps to the discrete time of the event, triggering the event, and calculating the next set of events. This type of simulation works particularly well with exponential arrival and departure distributions as these events bear the memoryless property, allowing each arrival and departure to be calculated separately without regard to the history of the system.

5.2 Inputs

The data collected in Section 4 was summarized in a series of `.csv` files serving different purposes.

- A `Network.csv` file held information regarding all queue nodes in the network. Each row contains QueueID, course level l, capacity c, frequency f, and centreID. These parameters was used to build the network within the simulation.
- A `Level.csv` file held information regarding the arrival proportions p_l and the discontinuing probability ρ_l for each level l. These parameters determined the level of arriving members and whether they would continue in the network.
- A `ImmigrationClass.csv` file held information regarding the immigration class proportions im_c and their mean service times μ_c. The proportions were used to determine the immigration class of an arriving member, and the overall mean service time is calculated as
$$\mu = \sum_{c \in \mathscr{C}} im_c \mu_c$$
.
- A `Properties.csv` file held information about the properties of the network, such as Arrival rate λ and the dropout probability χ.
- An assignment policy and a priority policy were defined, as described in Sections (3.5) and (3.6.1).

5.3 Outputs

Two output files were generated. The first contained the events of a queue node in the network. A row contained the time of **entry** to the node, the **queue wait time**, the time of **acceptance** into the course, the time spent **in course**, and the time of **leaving the class**. The second output file listed the **queue lengths** for every queue in the system at the beginning of each week simulated.

5.4 Calibrating the model

In order to produce results similar to the behaviour of the real-world system, the simulation needed to be calibrated by adjusting the flow in and out of the system. The incoming flow is controlled by the arrival rate λ, and the outgoing flow is controlled by the exiting probabilities ρ_l and χ (Section 3.7). When the system is at capacity, the length of the queues are proportional to the arrival rate λ; therefore, the adjusting the arrival rate λ affected the length of all queues in the

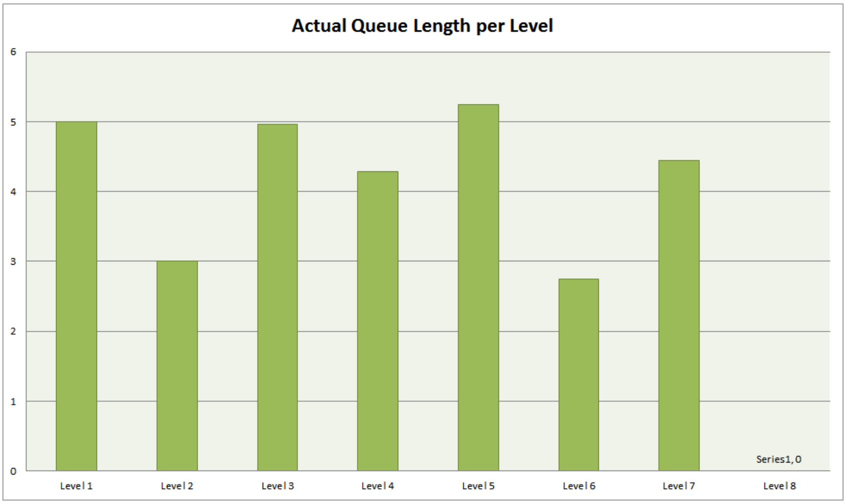

Figure 10: Actual Queue Length per Level

system. Similarly, increasing the discontinuing probability ρ_l for level l decreases the mean queue length for courses in level l, and increasing the dropout rate χ decreases the mean queue length of all queues except for those in level 1.

The simulation was calibrated in two phases using a *Uniform* assignment policy and a *FIFO* priority policy. First, the exiting probabilities ρ_l and χ were adjusting to obtain queues in similar proportions to the actual queues shown in Figure 10. Then, the arrival rate λ was adjusted until the correct queue lengths were obtained. The results of the calibration are shown in Figure 11.

5.5 Running the Simulation

The approach to running the simulation comes in two phases: solving the assignment policy, and solving the priority policy. With respect to the assignment policies, the three policies described in Section 3.5 are run in succession against the *FIFO* priority policy which is used as the standard case. For each policy simulation, the mean queue length, wait time, and total service time is recorded and analyzed to determine the most effective assignment policy, if any.

Once an optimal assignment policy is found that optimizes the flow of the system as a whole, we consider which of the priority policies described in Section 3.6.1 optimizes the flow of refugees with little effect to other immigration classes. Each priority policy is run against the optimal assign-

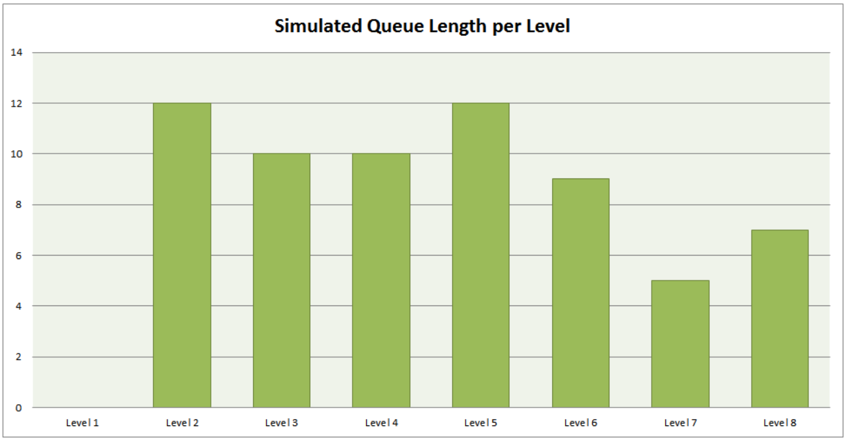

Figure 11: Simulated Queue Length per Level

ment policy from the previous phase. For each policy simulation, the mean queue length and total service times are recorded for the whole system, for refugees, and for other immigration classes. These measures are then analyzed to determine which is the most effective priority policy, if any.

Once the two phases are complete with respect to a stable flow, we run the two phases again with the presence of a simulated surge. In doing so, we seek to find solutions to the stable policy and the surge policy.

6 Solving: The Assignment Policy

The *surge* and *stable* optimal assignments are independently determined by performing two sets of simulations. In both sets a two year simulation is performed for each proposed assignment policy (Section 3.5) using the *FIFO* priority policy to represent the base case. The *Uniform* assignment policy is used to represent the current situation, and is the assignment policy against which the other two are compared. The mean queue lengths, wait times, and total service times for the levels are collected and examined to determine the optimal assignment for the surge and stable state. The simulated results in 12, and 13 shows that the *Deterministic Wait* assignment is expected to reduce the mean time in queues, queue length, and total service time in the system for both states. These findings suggest that the *Deterministic Wait* assignment policy is the optimal policy when the system is in a surge or stable state.

Assignment Policy	Wait Time	Total Service Time	Mean Queue Length
Uniform	1.9	19.3	2.5
Deterministic Wait	0.8	17.7	1.1
Deterministic Service	4.3	20.6	3.5

Figure 12: Effects of Assignment Policies During Stable Flow

Assignment Policy	Wait Time	Total Service Time	Mean Queue Length
Uniform	1.9	19.2	2.5
Deterministic Wait	0.9	18.2	1.3
Deterministic Service	3.4	20.1	4.3

Figure 13: Effects of Assignment Policies During Surge Flow

7 Solving: The Priority Policy

The optimal *priority policies* for the surge and stable states are determined using the state's respective optimal assignment policy. The optimal policy is determined by examining the effects of the policies on the mean total service time, and mean queue time for refugees relative to student of other *immigration classes*. For both the stable and the surge state, it has been determined in Section 6 that a *Deterministic Wait* assignment policy optimizes the network flow.

When the system is in a stable state, the simulated data suggest that the no priority policy will noticibly reduce the mean wait time, or total service time for refugees, Figure 14. With respect to the surge state, the simulation results 15 suggests that the *balanced policy* will lead to the largest reduction in total service time for refugees without notably increasing that time for other immigrants. These results suggests that the *Balance Policy* is the optimal priority policy during a surge state.

8 Surge and Stable Policies

We define a *policy solution* to the model as a pair of an assignment policy and a priority policy that optimize the flow of the network under certain conditions. The *stable policy* is a policy solution under regular flow conditions, which can be described as having a constant arrival rate λ

Priority	Wait Time	Wait Time for Refugees	Wait Time For Others	Total Service Time	Total Service Time for Refugees	Total Service Time for Others
FIFO	0.8	0.9	0.7	17.7	20.4	16.7
Balance	0.8	0.9	0.7	17.7	20.2	18.0
Refugee	1.0	0.8	1.0	18.1	20.4	17.1
Centre	0.8	0.7	0.9	18.0	21.6	17.2

Figure 14: Effects of Priority Policies During Stable Flow

Priority	Wait Time	Wait Time for Refugees	Wait Time For Others	Total Service Time	Total Service Time for Refugees	Total Service Time for Others
FIFO	0.9	0.8	1.0	18.9	21.5	17.8
Balance	0.9	0.9	0.9	18.0	20.2	17.3
Refugee	0.8	0.6	0.9	17.9	20.5	17.0
Centre	1.1	1.1	1.1	18.6	21.2	17.6

Figure 15: Effects of Priority Policies During Surge Flow

and constant immigration class proportions 4.3. When the arrival rate λ or any immigration class proportion p_c increases significantly, the *surge policy* takes effect. This policy solution is intended to optimize the flow of the network during a period of heightened arrival, with the intent of prioritizing the increased group until stable flow resumes and the stable policy is re-enacted.

As discussed in Section 6, the *Deterministic Wait* assignment policy is the best policy for reducing wait times and total service times, regardless of the accompanying priority policy. Based on these results, both the *stable policy* and the *surge policy* will be using the *Deterministic Wait* assignment policy.

In Section 7, it is noted that during a stable state there is no significant distinction between the different priority policies. In this regard, the optimal priority policy for the *stable policy* is any priority policy mentioned within this study. However, with regards to a surge period, it was concluded that a *Balance* priority policy optimized the flow of refugees. In fact, the pairing of a *Deterministic Wait* assignment policy and a *Balance* priority policy resulted in a reduction of the mean total service time by up to a week (Figure 14).

Based on the conclusions above, we define the *policy solutions* as follows:

$$stablepolicy = \{DeterministicWait, anyprioritypolicy\} \tag{3}$$

$$surgepolicy = \{DeterministicWait, Balance\} \tag{4}$$

9 Summary & Conclusions

The increase in Syrian refugees in Vancouver has caused a spike in demand for LINC courses, resulting in growing wait lists. The inefficiencies of the current LINC system, which allows students to enlist in as many courses as they wish, has lead us to propose a queueing network system with deterministic assignment of students to new courses (Section 3). By considering different assignment policies (Section 3.5), we sought to decrease the wait times for students, and reduce the variance between wait list lengths. In order to address the sudden surge of refugees, we explored different priority policies (Section 3.6.1) to prioritize the flow of refugees without significantly hindering the flow of others.

Upon conducting simulations it was found that the *Deterministic Wait* assignment policy — in which a member in the network chooses the queue node with the minimum expected wait time — produces the lowest expected wait times and total service times out of the policies considered. Furthermore, although no particular priority policy appeared to be more effective during normal flow, a *Balance* priority policy — in which members in the wait list exceeding the historical immigration class proportions are prioritized — resulted in meaningful decreases in expected wait times during surge periods.

10 Future Improvements

Although we have shown that using a deterministic assignment policy is ideal, the efficiency of such a system with regards to a real-world system may not be as clear since the assumption that members are unbiased with respect to which LINC centre locations they attend may not hold true. The deterministic policy relies on having students follow the directions output by the system, causing strain on the optimality of the network. However, the proportion of people needed to follow the system direction in order to guarantee improvement over a probabilistic system is a topic of further consideration that may confirm the system's viability in the real-world.

The priority policies considered in this study are very basic compared to those used by many systems, including the current LINC system. It is worth considering to further explore this topic, simulating priority policies with more complexities and layers which will also allow for a stronger comparison to existing systems. The current LINC system does not have a restriction on the number of queues a member may join. It may also be a point of consideration to find optimal assignment and priority policies under these conditions.

For this paper, we considered the narrow scope of Vancouver due to the ease of travel between centres. For future improvements, we would like to see the scope expanded to include all of Metro Vancouver. In reality, members from one city may choose to enroll in a centre in a neighboring city because the costs associated with a shorter wait outweigh the travel costs. Expanding the model to include a travel cost that factors in travel time, travel distance, bridge costs, and other factors in relation to two cities would help address this problem. Furthermore, a proper definition of regional zones based on LINC centre locations would be needed.

Finally, the assumptions made had all parameters equal for all members. Further data and research may suggest that certain groups may experience higher dropout rates, or perhaps lower mean departure times. Tuning the parameters to fit a set of classifications — such as sex, age, country of origin, mother tongue, etc. — would allow for a more accurate model and a better representation of real-world situations.

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