

Optimal Locations of Telecommunication Equipment: A Case Study for the City of Richmond, British Columbia, Canada ^{*}

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Abstract

Telecommunication service providers are competing in an environment where the pervasive nature of the mobile technology requires them to establish an improved network system with a wider coverage and a stronger signal strength to satisfy the escalating demand. The telecommunication equipment problem (TEP) is formulated as a combination of maximal coverage location problem (MCLP) and the assignment problem, and is solved using CPLEX, Excel, and R. The objective is to station devices in locations, where the largest number of consumers are able to enjoy desired services, while considering various demand factors, including: population density, points of interest, average household income, age, transit ridership, and the locations of existing towers.

^{*}Awarded Honourable Mention in the undergraduate paper competition at the 56th Canadian Operational Research Society Annual Conference in Ottawa, Canada.

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1 Introduction

The telecommunication equipment problem (TEP) is a distribution of a set of limited devices to a set of locations subject to a number of constraints in place. Considering the stiff competition among telecommunication firms, the growing number of users and rapid changes in technology, a well designed network is vital for success. Although cell towers (sites) play a vital role in creating these communication networks across various regions, there are a number of obstacles in constructing brand new towers. For instance, the discussion surrounding the recent news that whether Rogers Wireless should be permitted to install three new cell towers in West Vancouver [1], has brought a significant public attention to the topic. As a result, more recently, service providers are considering smaller, more flexible equipment to be installed on various existing fixtures i.e: street lighting poles and traffic lights.

In 2012, the City of Surrey introduced a one-year pilot program of installing such wireless transmission accessories on street light poles at the intersection of 152nd Street and 84th Avenue [2]. Comparing to the traditional telecommunication towers, these new devices have a number of advantages, making them attractive to all telecommunication firms. When constructing a tower there are many factors working in contrast to finding the best possible location; for example, the property owner may deny access, or the community may oppose such constructions in their neighbourhood; whereas, with the new approach, the equipment can be easily installed on the existing fixtures i.e. street lighting poles, and traffic lights, saving hundreds of thousands of dollars in construction costs, while expanding the number of potential locations. Furthermore, due to their size and design, the new cell equipment do not jeopardize the scenery of the neighbouring properties.



Figure 1: Example of the before (left) and after (right) attached telecommunication equipment. Adapted from Pilot Program for the Use of Street Light and Utility Poles for the Installation of Wireless Communications Infrastructure, by City of Surrey (2012). Retrieved February 27, 2014 from http://www.surrey.ca/bylawsandcouncillibrary/CR_2012-R186.pdf.

The aim of this study is to determine the best locations to place these devices to establish a network design that accommodates the largest number of consumers, thus leading to more subscription for the potential client, a telecommunication firm.

This study will focus on installing an improved network system using telecommunication equipment described above in the City of Richmond. A mixed-integer linear programming model including factors such as population density, age, income, transit ridership, places of interest and existing towers' coverage will be used to generate the optimal location for the equipment. Additionally, statistical analyses will be used to further improve the primary model, and the conclusion will be based on the comparison among a number of different scenarios.

1.1 Scope of Project

In this pilot study, the problem is restricted to an isolated area, the City of Richmond, south of Vancouver, due to concerns regarding the influence of other networks and the existing telecommunication towers and their coverage in neighbouring districts. As a result, the effects of the existing telecommunication equipment in the neighbouring districts are minimized. The City of Richmond is consist of most Lulu Island; however, a narrow part of the northeastern tip of Lulu Island, known as Queensborough, is part of the City of New Westminster [3]:

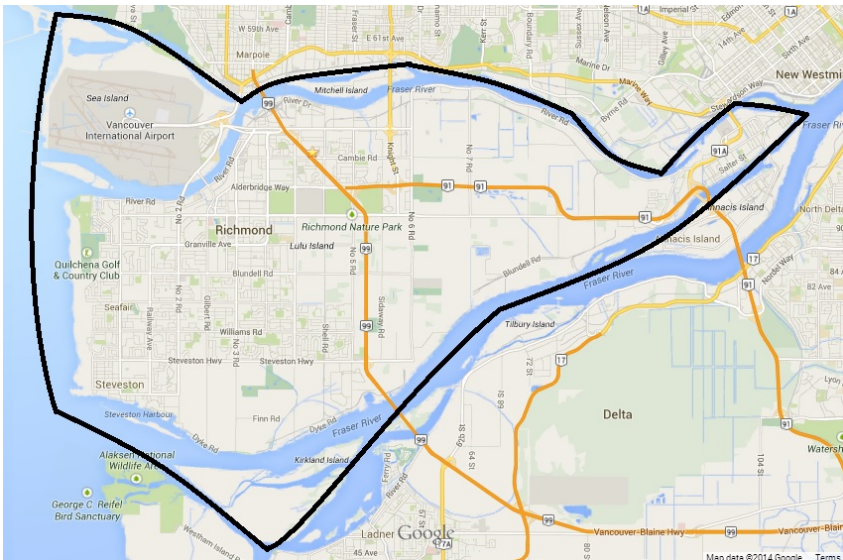


Figure 2: Boundaries of Lulu Island. Adapted from Richmond, British Columbia, Google Maps, 2014. Retrieved March 9, 2014 from <https://www.google.ca/maps/place/Richmond,+BC/@49.1718402,-123.1184624,11z/data=!3m1!4m2!3m1!1s0x54867599f4ef4d3d:0x6a502adba02fab5>.

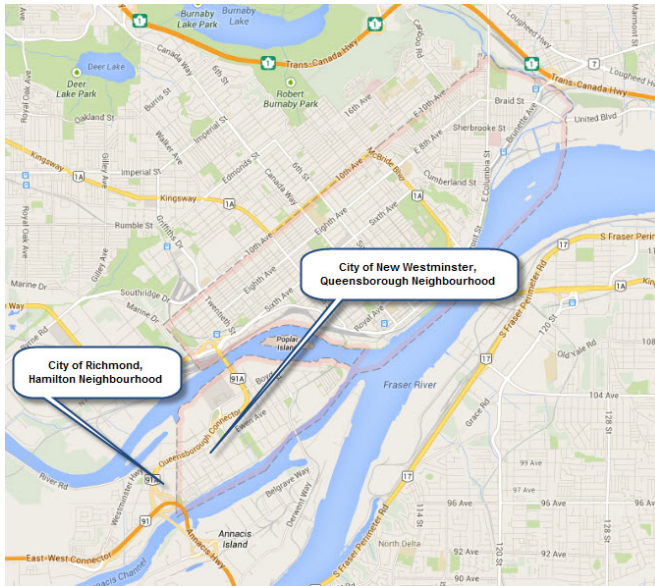


Figure 3: City of New Westminster Adapted from New Westminster, British Columbia, Google Maps, 2014. Retrieved March 21, 2014 from <https://www.google.ca/maps/place/New+Westminster,+BC/@49.2065565,-122.9175105,12z/data=!3m1!4b1!4m2!3m1!1s0x5485d8753ddb5097:0xeca8e918f64dfcee>.

Based on the information obtained from the City of New Westminster, it is determined that both adjacent neighbourhoods, Hamilton in the City of Richmond, and Queensborough in the City of New Westminster have similar characteristics [4]:

Population & Age Groups

	New Westminster		Queensborough	
Total population	57,850		5,485	
Age 0 to 4	2,925	5.1%	405	7.4%
Age 5 to 14	5,480	9.5%	740	13.5%
Age 15 to 24	6,410	11.1%	670	12.2%
Age 25 to 34	8,700	15.0%	945	17.2%
Age 35 to 49	15,795	27.3%	1,510	27.5%
Age 50 to 69	13,215	22.8%	990	18.0%
Age 70 to 79	3,140	5.4%	145	2.6%
Age 80 and above	2,195	3.8%	60	1.1%

Table 1: The Population and Age Groups, Queensborough Neighbourhood, City of New Westminster, 2011 Census. Retrieved, March 22, 2014, from http://www.newwestcity.ca/business/planning_development/demographics.php

For simplicity, the study region is referred to as the City of Richmond throughout the report.

2 Mixed-Integer Linear Programming Model

2.1 Approach

The TEP model is formulated as a mixed-integer linear programming (MILP), with the purpose of determining the best allocation of network equipment while considering various demand factors. Although there are many different factors that play a role in shaping the network demand, this study focuses merely on the following: population density, places of interest, age, income, transit ridership, and the existing telecommunication sites.

A grid of 24 rows by 37 columns is defined to cover the subject area; thus, there are 888 subsections each with the area of 0.36 km², (600 meters by 600 meters, approximately three city blocks):

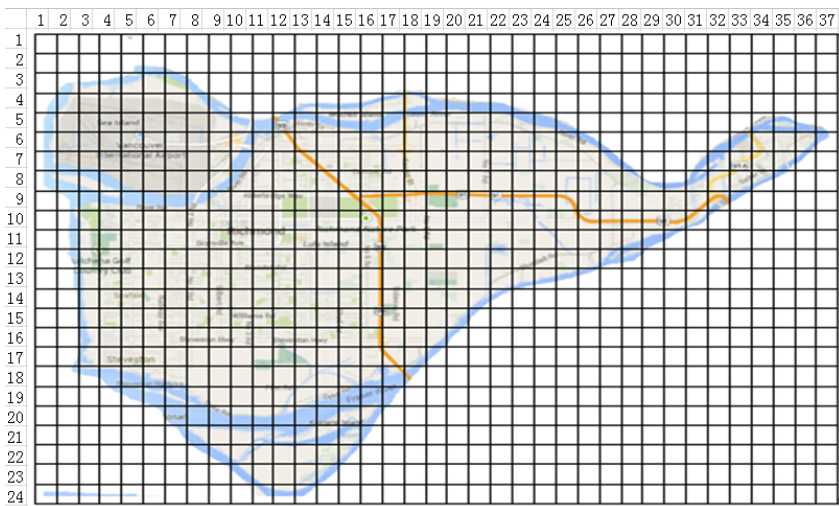


Figure 4: 24 by 37 Grid of the City of Richmond Adapted from Richmond, British Columbia, Google Maps, 2014. Retrieved March 11, 2014 from <https://www.google.ca/maps/place/Richmond,+BC/@49.1718402,-123.1184624,11z/data=!3m1!4b1!4m2!3m1!1s0x54867599f4ef4d3d:0x6a5024adba02fab5>.

Next, various sources are examined to obtain the required data for these six selected factors. The City of Richmond, the City of New Westminster, and Statistics Canada are the primary sources used in this report to collect the necessary information. In addition, TransLink Authority, and other third party sources (Pew Research Centre) are used to gather additional facts and figures.

2.2 Assumptions

1. This paper is tailored to meet the need of a new telecommunication provider, entering the local mobile market to establish an acceptable network coverage. In fact, this paper uses the data from the existing Wind Mobile network, a relatively new telecommunication company in Canada.
2. Each individual included in the study area owns a device and demands at least one of the services provided by a telecommunication service provider, i.e. phone calls, short messaging services (SMS), data, and etc.
3. Each grid cell, where inhabitable, is taken to have the same number of streetlights, and its center will be the location where to set up the telecommunication equipment.
4. For each device, the coverage is taken to be the maximum area of a square within the actual circular area:

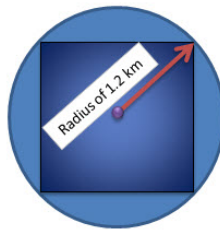


Figure 5: The network coverage for each of existing tower and the new equipment

5. Where there are two or more different scores for a specific grid cell, the most conservative figure is selected. As an example, suppose that a boundary line between two neighbourhoods cuts through a grid cell. Based on the two different population densities, the grid cell in question is scored as five and also seven (please refer to Figure 6). Under these circumstances, the priority is given to the factors with the higher score, i.e. the higher demand. This approach would ensure that the maximum demand is satisfied at all times.

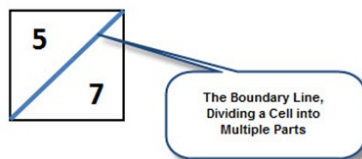


Figure 6: Boundary lines dividing a cell into multiple sections; the greater number, 7, is selected.

6. The problem is defined in a two-dimensional (2-D) environment, and it ignores the height. The range of coverage is sufficiently long enough to support high rises in both the high and low terrain of the City of Richmond.
7. The upper bound of the frequency transmitted by the antennas are far below the Health Canada guidelines [5]. Also, the frequency level will not cause any other harms and disruption towards the general public, local businesses, and travelling aircrafts.

2.3 Factors

Factors such as population density, places of interest, age, income, transit ridership and existing cell tower are taken into consideration. To ensure that these selected factors have no influences on one another, an independence test was conducted (Section 3.4).

2.3.1 Population Density

Considering the assumption that each individual owns a device and uses at least one of the services provided by a telecommunication service provider, highly populated areas would have a greater demand for the telecom equipment. As a result, the population densities of 14 planning areas across the defined region were obtained from Statistics Canada, through City of Richmond and City of New Westminister [6].

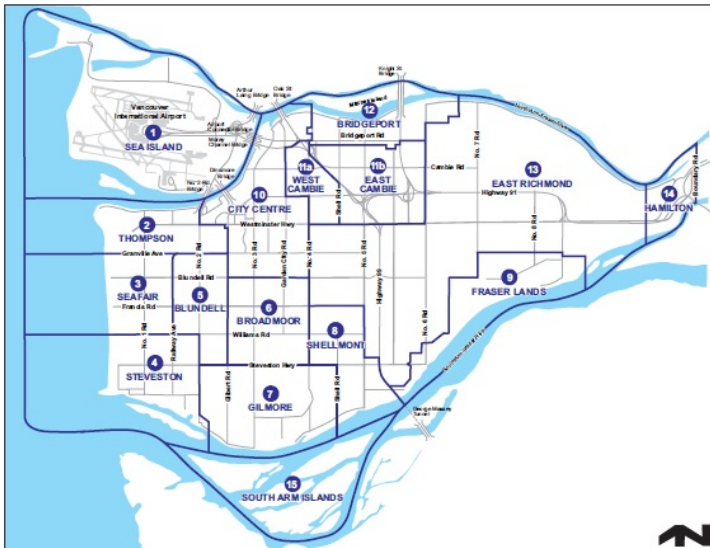


Figure 7: The 14 Planning Areas of City of Richmond. Adapted from Population Hot Facts, City of Richmond, 2013. Retrieved March 2, 2014 from http://www.richmond.ca/_shared/assets/Population_Hot_Facts_6248.pdf.

Population density is scored from 0 to 10, where the area outside the region are scored 0 (i.e. uninhabited), and the most populated area is scored 10. With more than 48,000 people, the City Centre is the most populated neighbourhood, and Gilmore with less than 500 people is the least populated. In Table 2, both East Cambie and Shellmont regions are near the median of 11,158; therefore, both regions are scored 5; other regions in between are scored based on the existing gaps among their population densities. Furthermore, the population density of City Centre, Shellmont, West Cambie, and Hamilton have significantly increased from 2006 to 2011. With the exception of the City Centre which already has the highest ranking possible, an additional point is added to all other three neighbourhoods to reflect the potential population growth. Therefore, the following scoring system is applied:

City of Richmond, Population Density	2006 Census	2011 Census	Percentage Change	Initial Score	Growth Factor	Final Score
City Centre	38,610	48,185	24.80%	10		10
Steveston	24,105	25,345	5.14%	8		8
Broadmoor	22,350	23,315	4.32%	8		8
Blundell	17,500	18,125	3.57%	7		7
Seafair	16,165	16,450	1.76%	6		6
Thompson	15,450	15,970	3.37%	6		6
Shellmont	10,000	11,130	11.30%	5	+1	6
East Cambie	10,400	11,185	7.55%	5		5
West Cambie	6,750	7,845	16.22%	4	+1	5
Hamilton	4,610	5,095	10.52%	3	+1	4
East Richmond / Fraser Lands	3,410	3,400	-0.29%	2		2
Bridgeport	2,995	3,190	6.51%	2		2
Sea Island	770	785	1.95%	1		1
Gilmore	450	460	2.22%	1		1
Outside of Richmond	-	-	0.00%	0		0
Total Population	173,565	190,480	9.75%			

Table 2: Population Density - Final Ranking, City of Richmond

There is another valid approach of scoring different neighbourhoods, one can divide all densities by the maximum number and multiply the results by ten; however, the inconsistent distribution of the residents among different neighbourhoods may lead to a biased conclusion. As an example, consider the top two populated areas, City Centre and Steveston. Applying this algorithm would score City Centre as a ten, whereas the second highly populated area would quickly drop to a five. Therefore, as explained above, a combination of ranking and scoring methodology is adopted.

2.3.2 Places of Interest

Although the population density is an indicator of the demand for various telecommunication services, it is solely based on the place of residence. Of course, one can expect that the actual density to be highly dynamic throughout the day when the general public is not at home, but rather at other places. Hence, other highly populated locations are taken into consideration as the places of interest. The following locations in the City of Richmond are considered in this study:

River Rock Casino and Resort, shopping centres (Richmond Centre, Aberdeen Centre, Lansdowne Centre), movie theatres, hospitals, libraries, educational institutions (Kwantlen Polytechnic University, CDI College, Omni College), supermarkets (Real Canadian Superstore, T&T Supermarket), IKEA, Richmond Auto Mall, summer night markets and Richmond Olympic Oval.

Three different categories are defined to group these locations based on their capacities, foot and vehicle traffic, and exposure: high demand, medium demand, and low demand. Next, taking the most conservative approach, i.e. considering the highest point of the demand throughout the year (the most crowded time) these interest points are distributed among the three different groups. For example, the Richmond Centre's busiest time on Boxing Day would fall into the high demand category, whereas a small library or a supermarket would have a relatively lower demand for various telecommunication services.

Relatively High Appeal - Ranking of 10	Relatively Low Appeal - Ranking of 2
River Rock Casino	Richmond Automall
Richmond Centre	Richmond Public Library - Cambie
Richmond Night Market	Richmond Public Library - Brighouse
Kwantlen Polytechnic University	Richmond Public Library - Shellmont
Airport - YVR	Richmond Public Library - Steveston
	SilverCity River Port Cinema
	Richmond Hospital
	Richmond Hospital Auxiliary
	IKEA
	Skytrain Stations

Relatively Medium Appeal - Ranking of 5
Aberdeen Centre
Lansdowne Centre
New Westminster Smartcentre
CDI College
Omni College
Costco
T&T
Richmond Olympic Oval

Table 3: The Scoring System for Points of Interest

2.3.3 Age

Various studies have indicated that smartphone ownership and the data usage vary across different age groups. In January 2014, a research survey conducted by Pew Research Centre [7] concluded that the smartphone ownership of different age groups with a sample size of 1,006 adults.

Consequently, the areas with a higher percentage of people aged between 15 to 64 are allocated with a greater weight. The data obtained is sorted from lowest to highest for each area, and a scoring system from 0 to 10 is assigned correspondingly, where areas outside the City of Richmond are scored 0. A score of 10 is assigned to the highest percentage (79%), 5 to the median (72%), and 1 to the lowest (54%).

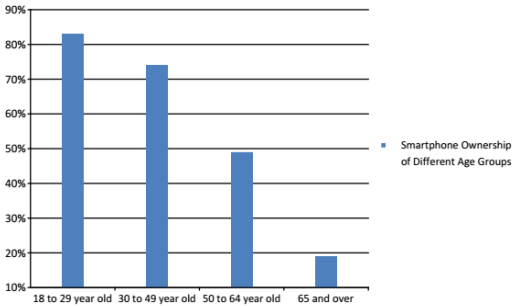


Figure 8: Smartphone ownership of different age group

2.3.4 Income

The scatter plot of GDP and Phone Penetration in Appendix B indicates that those in higher income brackets with more disposable income are able to purchase more devices and services from telecommunication companies, who have a lower tolerance of service disruption. From 2006 Census report provided by Statistics Canada, average household net income in each neighbourhood can be scored through 0 to 10: while 1 represents the lowest net income, \$12,970, 5 characterizes the medium net income of \$22,643, and 10 for the highest net income of \$32,319. The area outside of the city boundary is scored a zero for evident reasons. Figure 9 shows the output of the scoring system for the average household income data:

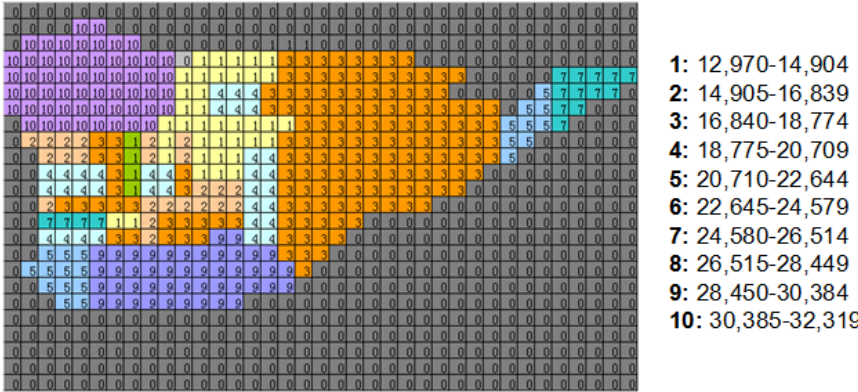


Figure 9: Scoring system applied to net income

2.3.5 Transit Ridership

The number of transit riders have increased since the opening of the Canada Line skytrain. According to the transit ridership report by TransLink, the total passenger boardings for Canada Line from January to June 2011 was 19,185,093, and the average weekday boardings for Canada Line in June 2011 was 136,259 [8]. With this in mind and considering the assumption that every transit rider carries at least a mobile device, the demand for a stable cellular network system is quite evident. The transit ridership data contains the percentage of the Richmond's population who travel on Translink via bus and skytrain. Similar to the previous factors discussed above, a scoring system of zero to ten has been adopted to reflect the transit ridership, where areas without a transit route are scored a zero. After analyzing the data, it is concluded that the ridership ranges from 6% to 22% with the median of 12%; hence, 1 is allocated to the minimum, 5 to the median, and 10 to the maximum.

2.3.6 Existing Cell Sites

Taking into account the existing towers would help in creating a more realistic scenario, under which the coverage range as well as the capacity of each tower would directly influence the objective function. Considering various service providers across the region, the existing locations of Wind Mobile towers are reflected in this study. This specific provider is selected due to the fact that it has recently entered the Canadian market, and when compared to the more established networks (Telus, Bell, and Rogers), its network is premature. Additionally, all the existing towers are well spread out throughout the defined study region. Each tower provides the same frequency of 2100 MHz, which translates into an area of approximately 5 km², (radius of 1.2 km). It must be noted that the area defined in this report is a subjective discrete integer value. Under different scenarios, the actual coverage-area depends on the hardware in the equipment cabinet, base station (BTS), the technology (i.e. 3G, 4G LTE, UMTS), and the network configurations provided by the service provider.

2.4 Independence Test

A significance test of independence [9] was applied to all different factors considered in this study: population density, transit ridership, income, age, places of interests, coverage of existing tower and the capacity covered by of the existing towers. Based on the analysis, it is determined that the correlation between the coverage of the existing towers ($E_{i,j}$) and the capacity covered by existing tower ($F_{i,j}$) is 1, indicates that one factor can be fully predicted by another; consequently, including both predictors could cause biased results. Moreover, other factors have a correlation of less than 0.8, which means it is reasonable to include population density, transit ridership, income, age, places of interest and the capacity covered by existing tower in the objective function. However, there is a probability that such factors may have a joint effect on the objective function. To further examine this issue, another statistical significance test such as an analysis of variance (ANOVA) can be applied after obtaining the optimal solution for telecommunication equipment's location.

2.5 Data Collection Method

As explained previously, various sources are used to extract the data required to test the models developed in this paper.

1. **Population Density** is obtained through Statistics Canada, and it is based on 2011 Census, while considering 2006 Census and the percentage of change between the two. Although according to the City of Richmond's projections through Urban Futures Inc, as of March 2013, the total population is estimated at 205,000 people (an increase of 8% since 2011 Census), the distribution among different neighbourhoods remains relatively stable.
2. **Points of Interest** are based on the knowledge of the local area and the latest redevelopment projects in the City of Richmond. The facilities' capacities are obtained through their websites or through various third party sources. Please refer to Appendix A, which exhibits further details on the most attractive points of interest: Vancouver International Airport, Richmond Night Market, Richmond Centre (Mall), River Rock Casino Resort, and Kwantlen Polytechnic University.
3. **Transit Ridership** is built upon the 2006 Census with the percentage of transit riders in different areas of the City of Richmond. However, the the number of transit riders has increased since 2006 due to new public transit routes, increase in fuel prices, 2010 Winter Olympics, as well as the Canada Line SkyTrain that started to operate in August of 2009. According to TransLink, in 2009 [10], the majority of improved transit services and travel routes are between the points of interests (i.e. YVR Airport to Richmond Centre); thus the trend of transit ridership across the 14 planning areas of the City of Richmond from the 2006 Census and 2011, remains relatively similar.
4. Both **Age** and **Income**, are based on the 2006 Census due to the fact that the City of Richmond uses 2006 Census to form its planning maps [11]. Also, a common concern would be that the data might be outdated, and it does not represent today's marketplace. However, where the data is available through other sources, the comparison between 2006 and 2011 information indicates that the distribution among different age groups and income brackets remains relatively stable.
5. As discussed earlier, Wind Mobile's existing network is mirrored in this study. When analyzing the **Existing Telecommunication Devices** three questions that must be answered:
 - (a) the location of existing towers and cell equipment,
 - (b) the area covered by each equipment, and
 - (c) the maximum number of consumers that can benefit from provided services.

In Figure 10, Canadian Cellular Towers Map [12], which maps the existing towers is used to answer the first question. Also, as explained in §2.3.6 the radius of coverage has been

determined to be 1.2 km. A well established network, Rogers Communications, is considered to answer the last question. Moreover, Rogers has distributed its equipment across 45 different locations to support the service level demanded by its subscribers.

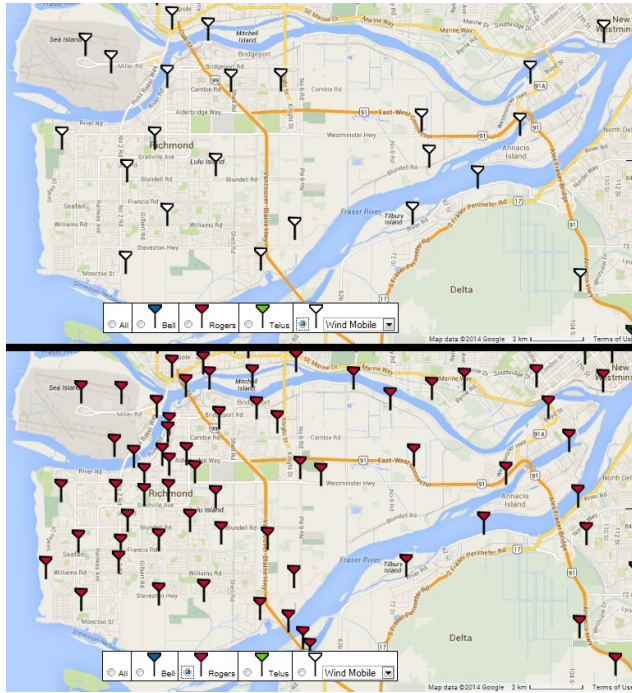


Figure 10: The Existing Sites, Wind Mobile vs Rogers Communications Adapted from Canadian Cellular Towers Map. Retrieved March 5, 2014 from http://www.ertyu.org/steven_nikkel/cancellsites.html.

Although the people over the defined region (population of 225,000) have various other options to choose as their service providers, Rogers only services a portion of the total population. In the absence of specific information and in order to have the same reference point, it is assumed that the 45 towers can satisfy the maximum potential demand. Therefore, it is concluded that on average each tower is capable of satisfying 5,000 clients. ($225,000 \div 45 = 5,000$). Considering each equipment covers nine grid cells, it would approximately be 560 clients per cell ($5,000 \div 9 = 556$), assuming the potential clients are evenly distributed among the nine grid cells; however, in reality this would not be the case, and the distribution among different grid cells would not follow a uniform distribution. Therefore, to form a more realistic value, the maximum demand in a grid cell is divided by the maximum number of towers covering that grid cell.

Based on the data obtained, at their peaks, both Vancouver International Airport and Richmond Night Market (Appendix A) are the most populated points across the study area, where the demand for telecom services is maximized:

Points of Interest	Number of Visitors per Day, at Peak	Number of Grid Cells Covered by the Facility	The Maximum Demand per Grid Cell
Vancouver International Airport	60,000	6	10,000
Richmond Night Market	30,000	3	10,000

Table 4: The maximum potential demand per each cell

Using Rogers Communications' existing network as the reference point, it is determined that the maximum number of devices covering the Richmond Night Market is three.

Thus, it is concluded that at each grid cell, approximately, the maximum of 3,500 (10,000 \div 3) subscribers' demands can be satisfied, and a cell that is covered by three towers has fully met its demands. Hence, a factor of -3.5 (10 \div 3 = 3.33) is applied for the coverage provided to each cell. In other words, if a cell is covered by three towers, then its maximum demand can be met.

3 Formulation

All the factors in §2.3, are scored from 0 to 10 depending on how greatly they would influence the network demand:

3.1 Coefficients

For $i, j \in \mathbb{Z}$ and $i = 1, 2, \dots, m$ rows and $j = 1, 2, \dots, n$ columns:

A_{ij} = Age's score from 15 to 64 years old at (i, j) :	$A_{ij} = [0, 10]$
E_{ij} = Existing Wind Mobile towers locations at (i, j) :	$E_{ij} = [0, 1]$
F_{ij} = Existing Wind Mobile towers capacity at (i, j) :	$F_{ij} = [-10, 0]$
I_{ij} = Points of interest's score at (i, j) :	$I_{ij} = [0, 10]$
P_{ij} = Population density's score at (i, j) :	$P_{ij} = [0, 10]$
T_{ij} = Public Ridership's score at (i, j) :	$T_{ij} = [0, 10]$
W_{ij} = Income's score at (i, j) :	$W_{ij} = [0, 10]$
H = The maximum number of equipment in each grid cell	$\forall(i, j)$
G = The maximum number of equipment in adjacent grid cells	$\forall(i, j) \in (-1, 1)$
S = The supply of telecommunication equipment (antenna & cabinet) provided by the telecommunication firm.	

3.2 Variables

For $i, j \in \mathbb{Z}$ and $i = 1, 2, \dots, m$ rows and $j = 1, 2, \dots, n$ columns:

x_{ij} = Allocation of new equipment in each grid cell at (i, j)

c_{ij} = Coverage of new equipment in adjacent grid cells at (i, j)

l_{ij} = Coverage of existing and proposed new equipment at (i, j)

k_{ij} = Number of existing and proposed new equipment at (i, j)

3.3 Constraints

1. Binary Decision Variables:

$$x_{ij} = \begin{cases} 1, & \text{allocate telecommunication equipment at } (i, j) \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ij} \in \mathbb{B}$$

2. Supply:

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij} \leq S; \text{ All equipment allocated cannot exceed the supply } S \text{ where } S \in \mathbb{Z}$$

3. Coverage at corners, sides and middle (defined in a 3 by 3 grid):

for $i = 1, j = 1$ $\sum_{i=1}^{i+1} \sum_{j=1}^{j+1} x_{ij} = c_{ij}$ $\sum_{i=1}^{i+1} \sum_{j=1}^{j+1} k_{ij} = l_{ij}$	for $i = 1, 1 \leq j \leq n$ $\sum_{i=1}^{i+1} \sum_{j=1}^{j+1} x_{ij} = c_{ij}$ $\sum_{i=1}^{i+1} \sum_{j=1}^{j+1} k_{ij} = l_{ij}$	for $i = 1, j = n$ $\sum_{i=1}^{i+1} \sum_{j=1}^j x_{ij} = c_{ij}$ $\sum_{i=1}^{i+1} \sum_{j=1}^j k_{ij} = l_{ij}$
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4. Coverage of new equipment cannot exceed G

$$c_{ij} = \begin{cases} G & \text{Proposed coverage at } (i, j) \\ 0, & \text{otherwise} \end{cases}$$

5. Proposed coverage of new equipment

The proposed coverage of new equipment should not exceed the maximum amount of equipment in adjacent grid cells.

$$l_{ij} \leq G \quad \forall (i, j)$$

6. Proposed location of new equipment

The proposed location of new equipment should not exceed the maximum amount of equipment in adjacent grid cells.

$$E_{ij} + x_{ij} = k_{ij} \leq k_{ij}$$

7. Proposed coverage of new equipment

The proposed coverage of new equipment should not exceed 2.

$$G = [1, 2] \text{ in this study}$$

8. Proposed location of new equipment

The equipment at each grid cell should not exceed 1.

$$H \leq 1 \quad \text{for } H \in \mathbb{Z}_+$$

3.4 Statistical Analysis

To complete the following statistical analyses, R, a statistical software, is used to tests demand factors in the objective function.

3.4.1 Significance Test

In order to apply a statistical significance test, CPLEX is used to simulate feasible solutions through locating telecommunication equipment by setting up the equipment supply, S , equals to 50, 100, 150, 200, 250, and 300. By treating the objective function as a multiple regression model, a statistical significance test [9] is applied to factors such as population density, transit ridership, income, age, existing tower capacity and points of interest to test each factors' significance. Table 5 shows the significance of each factor, in which age and transit ridership have minor influence on the model.

Significance Table (none, low, medium, high)						
	S = 50	S = 100	S = 150	S = 200	S = 250	S = 300
Population	high	high	high	high	high	high
Transit	none	medium	medium	none	low	high
Income	high	high	high	high	high	high
Age	none	low	high	none	none	high
Interest	high	high	high	high	high	high
Existed Tower	high	high	high	high	high	high
Standard Error	0.1716	0.2039	0.2384	0.2412	0.2405	0.2423

Table 5: Significance Test on each factor under six supply scenarios

To determine which factor (transit and/or age) should be excluded from the model, an analysis of variance (ANOVA) test, taking $S = 200$ as an example, is applied to following models: the primary model, including both age and transit, and other factors; the model including only excluding transit ridership; the model only excluding age; and the model excluding both factors. The result illustrated in Table 6 suggests that both factors must be dropped from the objective function in order to achieve a model with a higher accuracy level. The Akaike Information Criterion (AIC) model selection method generated the same model (the revised model — excludes both transit and age), since it has the highest AIC value. The outcome of both ANOVA test and AIC selection method are shown below:

Significance Test Result (S = 200)				
	Original Model	Model excludes transit	Model excudes age	Model excludes both
p-value	-	0.16	-	0.02
Significance	none	none	none	medium
AIC	-3128.51	-3125.4	-	-3127
Standard Error	0.2412	0.2414	0.2414	0.2419

Table 6: Multiple comparison ANOVA test and AIC on four models

3.4.2 Coefficient Test

The objective function treats each factor with the same weight, 1; however, one factor could affect the model more than the other; for instance, the population density may have a greater influence on the output than the income factor. Therefore, a statistical analysis for assigning coefficients to each factor is applied to the data with the supply number, S , equals to 50, 100, 150, 200, 250, and 300. The result in Table 7 shows places of interest has the heaviest weight on the objective function, and as the sample size increases, population density would have the most impact. It is reasonable to believe that these factors (population, income, places of interest, existing towers) should not all have the same weight as 1, and further research will be required to determine the coefficient of each factor.

Coefficient Test						
	S = 50	S = 100	S = 150	S = 200	S = 250	S = 300
Population	0.033	0.07	0.065	0.081	0.085	0.087
Income	0.009	0.011	0.045	0.056	0.068	0.075
Interest	0.072	0.075	0.07	0.052	0.034	0.027
Existed Tower	0.031	0.045	0.055	0.055	0.036	0.046

Table 7: Coefficient Test on each factor with different equipment supply numbers

3.5 Objective Function

3.5.1 Primary Model:

$$\text{Maximize } \sum_{i=1}^m \sum_{j=1}^n (P_{ij} + W_{ij} + A_{ij} + T_{ij} + I_{ij} + F_{ij})c_{ij}$$

3.5.2 Revised Model:

$$\text{Maximize } \sum_{i=1}^m \sum_{j=1}^n (P_{ij} + W_{ij} + I_{ij} + F_{ij})c_{ij}$$

3.6 CPLEX and Excel

CPLEX and Microsoft Excel are used to implement and test the models developed in this paper. All the data collected are stored into one Excel file with multiple spreadsheets for an easy access. CPLEX reads these spreadsheets to the data file (.dat) and the model file (.mod) calls these data into the model. Next, CPLEX Optimizer builds and solves the MILP model and inputs the results to the Excel worksheet, where the analyses is conducted. Furthermore, the model-fit function and Analysis of Variance (ANOVA) table are used to test significance on factors (population of density, income, age, transit ridership and existed tower capacity), and Akaike Information Criterion (AIC) model selection method is used to modify the objective function.

4 Results

Optimal locations to allocate communication equipment generated from both the primary and revised MILP model are summarized. Several comparisons between models under different scenarios are analyzed and interpreted.

4.1 Primary Mixed-Integer Linear Programming Model

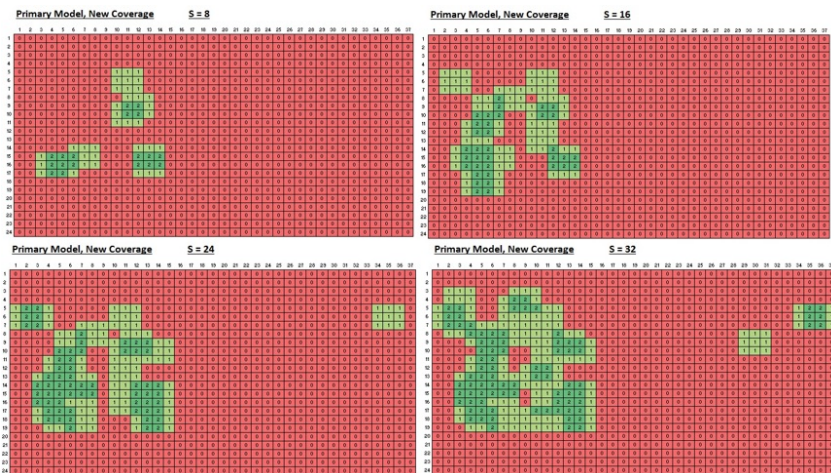


Figure 11: Coverage from new cell equipment (c_{ij}) in the primary model for $S = 8, 16, 24,$ and 32

It is clear from Figure 11, that the majority of the clusters of telecommunication equipment are allocated to the City Centre and its nearby populated neighbourhoods. Under the scenario where there are 8 devices available, with the maximum number of equipment at each cell equal to 1 and the maximum number of coverage by neighbouring devices set to 2, the objective function, optimal network utilization, is 1756. Furthermore, the optimal network utilization for supplies of 16, 24, and 32 equipment are 3257, 4616, and 5881 respectively. As shown below, 24 cell equipment are required to cover most of the network demand from southern Sea Island to the intersection of Number 2 Road and Steveston Highway. Additionally, as the supply of the new equipment increases, the network coverage expands to the eastern side of the region to cover both Hamilton and Queensborough neighbourhoods.

4.2 Revised Mixed-Integer Linear Programming Model

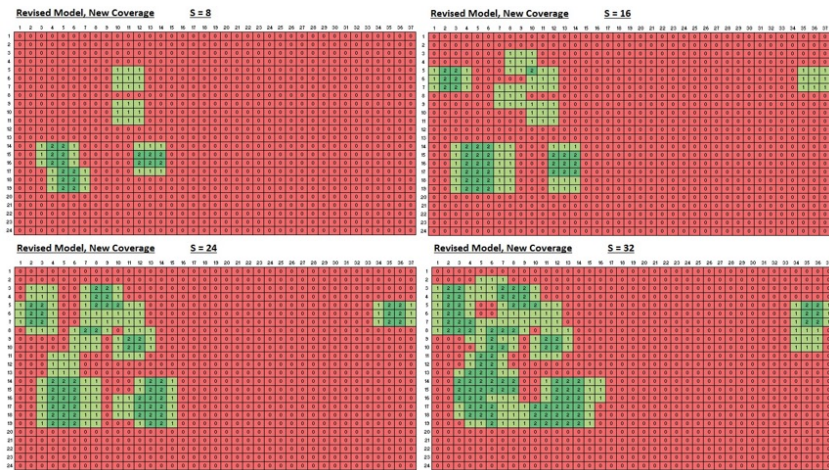


Figure 12: Coverage from new cell equipment (c_{ij}) in the revised model for $S = 8, 16, 24,$ and 32

In the revised model, the coverage of optimal solution for locating equipment in Figure 12 clearly shows that the demand in most of the western parts of the study region are covered, and as the supply number of telecommunication equipment increases, a small portion in the eastern area will also be covered. The revised model provides coverage to the eastern section of the city at an early stage (i.e. supply of 16 equipment) and expands more into the south east areas. The optimal network utilization and percentage of coverage for supplies of 8, 16, 24, and 32 equipments are 972 (17%), 1775 (32%), 2509 (47%), and 3164 (63%) respectively.

4.3 Comparison Between the Primary Model and Revised Model

The statistical analysis of significance test on factors suggested that the revised model could generate a more accurate result compared to the primary model. CPLEX computed both models with different equipment supply numbers $S = 8, 16, 24$ and 32 , and produced the following results in Table 8. There are strong indications to conclude that the revised model is better than the primary model, since the revised model can meet more demands and have fewer overlapped areas. In addition, the generated optimal solution from the revised model locates telecommunication equipment both on west and east sides in the City of Richmond, which provides an improved network for the high demand at the City Centre, and also covers the low demand at the east side. Thus, the revised model is sustained.

Comparison Table								
	Primary Table				Revised Model			
Supply	S = 8	S = 16	S = 24	S = 32	S = 8	S = 16	S = 24	S = 32
Overlap	22	49	76	103	24	46	75	100
Demand met	852	1635	2297	2971	972	1775	2509	3164
Coverage %	18.91	28.7	38.48	48.26	18.48	29.35	38.7	48.91

Table 8: Comparing between the Primary Model and Revised Model

4.4 Comparison Between Wind Mobile and New Cell Equipment

Wind Mobile currently has located 18 telecommunication towers in the City of Richmond, which covers 33% of consumers' demands. By not taking these towers into account, the revised model with the same supply equipment amount S equals to 18, sets at most two equipment in adjacent grid cells ($c_{ij} = [0, 2]$), and generates the optimal solution. The coverage of Wind Mobile and the proposed location of the optimal solution are presented in Figure 13 as below:

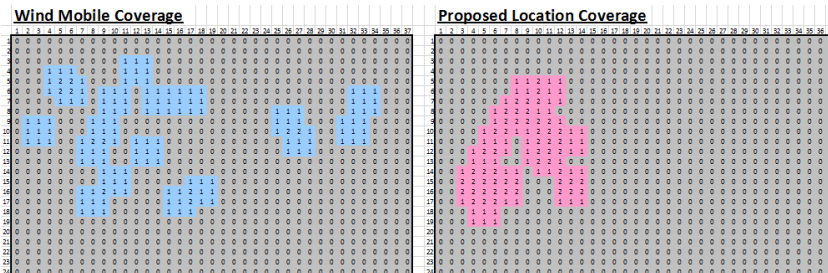


Figure 13: Wind Mobile & Proposed Location coverage with $c_{ij}=[0, 2]$ and $c_{ij} \in \mathbb{Z}$

The proposed location obtained from the model suggests to locate telecommunication equipment at east side (City Centre) of the grid, due to high demands of population density, income and places of interests there. The optimal network utility with the proposed location can be improved to 3608, while Wind Mobile currently meets network utility of 1577. However, since the model

allows two telecommunication equipment to locate near each other, there exist more overlapped areas at the proposed network coverage compared to the Wind Mobile network. To reduce such coverage overlaps, another optimal solution is generated from the revised model to allow only one equipment being installed in 9 grid cells. Figure 14 shows the results obtained from CPLEX.

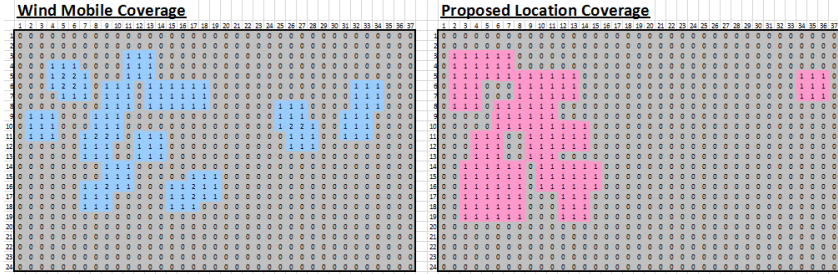


Figure 14: Wind Mobile & Proposed Location coverage with $c_{ij} = [0, 1]$ where $c_{ij} \in \mathbb{Z}$

Comparison Table			
	Wind Mobile	Proposed Location (max = 2)	Promosed Location (max = 1)
Supply	18	18	18
Overlap	11	53	0
Demand met	1517	3608	3249
Coverage %	32.83	23.7	33.48

Table 9: Comparison among the Wind Mobile, Proposed Location with $c_{ij} = [0, 2]$, and Proposed Location with $c_{ij} = [0, 1]$ where $c_{ij} \in \mathbb{Z}$

Both Figure 14 and Table 9 demonstrate that the model ($c_{ij} = [0,1]$) generate a better optimal location choice while setting the same amount of equipment supply S equals to 18, because the proposed location satisfies more for Richmond network users, and has fewer overlapped areas. Furthermore, results show that the proposed location with zero overlaps increases the network coverage by 10% by controlling the overlapped areas. Nonetheless, existing towers often have an important influence on a long-term decision making and cannot be neglected. To improve current coverage percentage based on Wind Mobile network, optimal solution of locating telecommunication equipment under different scenarios are brought into comparison.

4.5 Scenarios of Controlling Overlaps

It is observed that controlling overlapped coverage areas (setting telecommunication equipment locating near each other or not) can affect the decision making of this problem. Thus, ten optimal solutions were generated by the revised model with equipment supply amount S equals to 8, 16, 24, 32, and 52 and under two scenarios: allowing two cell equipment locating closely

($c_{ij} = [0, 2]$); and not allowing any overlapped coverage area ($c_{ij} = [0, 1]$). Table 10 summarises differences between two scenarios as following:

Comparison Table								
	$c_{ij} = 0, 1$				$c_{ij} = 0, 1, 2$			
Supply	S = 8	S = 16	S=24	S = 32	S = 8	S = 16	S=24	S = 32
Overlap	0	0	0	0	24	46	75	100
Demand met	891	1602	2151	2531	972	1775	2509	3164
Coverage %	15.65	31.3	46.96	52.61	18.48	29.35	38.7	48.91

Table 10: Comparison among the Wind Mobile, Proposed Location with $c_{ij}=[0, 2]$, and Proposed Location with $c_{ij}=[0,1]$ where $c_{ij} \in \mathbb{Z}$

By controlling overlapped areas as zero, the demand met decreases, while the coverage percentage increases a lot. Furthermore, adding every additional equipment in the model (no overlaps) increases the coverage percentage by 6%, while the other model (allowing two equipment nearby) increases only 2% on coverage percentage. In fact, 52 towers can provide a 100% network coverage for the City of Richmond (Figure 15). In summary, by allowing two cell equipment to allocate near each other will provide services to meet high user demands while controlling overlapped areas can increase coverage percentage. Hence, both scenarios should be considered for choosing suitable telecommunication equipment locations.

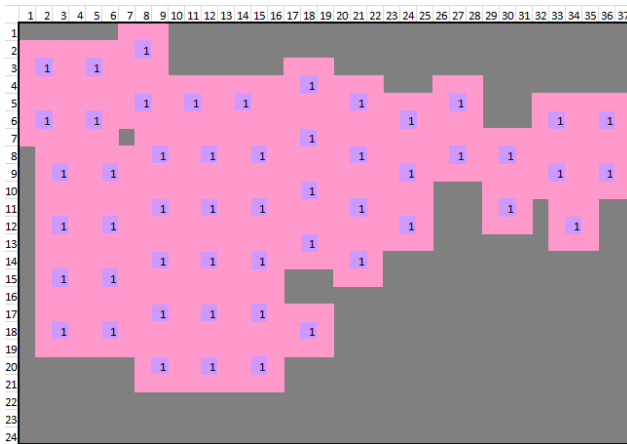


Figure 15: Wind Mobile & Proposed Location coverage with $c_{ij}=[0, 1]$ where $c_{ij} \in \mathbb{Z}$

5 Weighted Revised Model

Weights on the factors in the revised model are applied to the current scoring system due to the fact that a significantly larger study region would be extremely difficult to partition and

to categorize the data into sensible intervals. Considering the recent study conducted by Pew Research Centre, education and age seem to have less impact on the smartphone ownership and the internet usage [13]. Figure 16 compares a notable increase in income brackets of smartphone ownership between 2011 and 2012. Furthermore, considering that the cost of purchasing both a smartphone and a plan are increasing [14], it is reasonable to interpret that the age and education of each individual would demand the services provided by telecommunication firms. As previously described (Section 3.4), age and transit in our revised models are omitted in the weighting system.

Smartphone ownership demographics

*% of American adults age 18+ within each group who own a smartphone.
 "Smartphone ownership" includes those who say their phone is a smartphone, or who describe their phone as running on the Android, Blackberry, iPhone, Palm or Windows platforms.*

	May 2011	Feb. 2012
All adults (age 18+)	35%	46%
Men	39	49
Women	31	44
Race/Ethnicity		
White, non-Hispanic	30	45
Black, non-Hispanic	44	49
Hispanic (English- and Spanish-speaking)	44	49
Age		
18-29	52	66
30-49	45	59
50-64	24	34
65+	11	13
Household Income		
Less than \$30,000/yr	22	34
\$30,000-\$49,999	40	46
\$50,000-\$74,999	38	49
\$75,000+	59	68
Education level		
No high school diploma	18	25
High school grad	27	39
Some college	38	52
College+	48	60
Geographic location		
Urban	38	50
Suburban	38	46
Rural	21	34

Figure 16: Smartphone ownership demographics. Adapted from Pew Research Centre. Retrieved March 31, 2014 from http://www.pewinternet.org/files/old-media/Files/Reports/2012/PIP_Digital_differences_041312.pdf.

For each factor, the weighting method divides each score proportional to the maximum possible demand and multiply this proportion by 10.

1. Population Density

The most populated location, City Centre, is home to 50,000 residents across 40 grid cells,

which translates to 1,250 points of demand at each grid cell. Table 11 shows the population density in each neighbourhood and their respective scores. respective scores.

City of Richmond, Population Density	2011 Census	Scores
City Centre	45,155	10
Steveston	25,345	6
Broadmoor	23,315	5
Blundell	15,125	3
Seafair	16,450	4
Thompson	15,970	4
East Cambie	11,155	2
Shellmont	11,130	2
West Cambie	7,545	2
Hamilton	5,095	1
East Richmond / Fraser Lands	3,400	1
Bridgeport	3,190	1
Sea Island	755	0
Gilmore	460	0
Outside of Richmond	0	0
Total Population	179685	-

Table 11: Weighted score table for the population density

2. Points of Interest

As shown in Table 4 (Section 2.5), both Vancouver International Airport and Richmond Night Market can accommodate up to 10,000 people in each cell.

3. Income

Since Sea Island, the home to the Vancouver International Airport, is an outlier of relation between the net income and the population density, thus Steveston's Village with a net income of \$21,887 is selected as the maximum. The corresponding population density at the same area is 24,105. By dividing the population density by the covered 58 grid cells gives the points of demand, which is 416.

4. Existing Towers

As discussed previously under §2.5.5, considering Rogers Communications' existing network, it is determined that three devices are used to cover the maximum potential demand of 10,000 at the annual Richmond Night Market. As a result, a factor of -3.5 is applied to the existing tower to reflect the existing towers and their influence in meeting the potential demand.

Factors	Maximum Potential Demand Points	Coefficients
Population Density	1,250	1.25
Points of Interest	10,000	10
Existing Towers	-3,500	-3.5
Income	416	0.42

Table 12: Maximum potential demand points and corresponding coefficients for the factors

Therefore, the weighted revised objective function changes to:

$$\text{Maximize } \sum_{i=1}^m \sum_{j=1}^n (1.25P_{ij} + 0.42W_{ij} + 10I_{ij} - 3.5F_{ij}^*)c_{ij},$$

where F_{ij}^* is the existing towers in adjacent cells = $[0, 2]$

In the weighted revised model, both points of interest and existing Wind Mobile towers have heavy influences on the result. On the contrary, population density and income have low impact, which may result from the popularity of advanced devices and their affordable prices, and the joint effect of two factors: places of interests and population density. CPLEX and OpenSolver is used to generate the optimal solutions for the weighted revised objective function, with the proposed coverage l_{ij} sets to be 2, and the proposed location k_{ij} sets to be 1. In Figure 17, new equipment are mainly located around the City Centre, where there are high demand points. In comparison to the unweighted revised model, the coverage percentage decreases in some areas to increase coverage overlapping for meeting high demands.

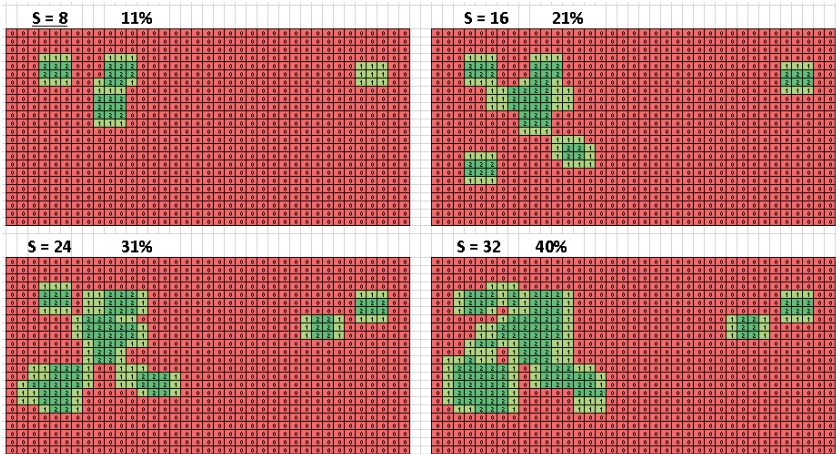


Figure 17: Coverage of the proposed coverage of new equipment (l_{ij}) and the proposed location of new equipment (k_{ij}) in the revised model for $S = 5, 10, 15,$ and 20 .

While considering Rogers Wireless' existing tower locations, the maximum number of existing towers in a grid cell (k_{ij}) is 2 and the maximum coverage by existing Rogers' tower (l_{ij}) is 5. In Figure 18, the optimal coverage by new equipment is significantly lower than the previous Wind Mobile's coverage. However, the majority of the coverage satisfies the demand in the high populated areas with 24 telecommunication equipment before allocating coverage to Steveston, and parts of East Richmond and Hamilton with 32 equipment.

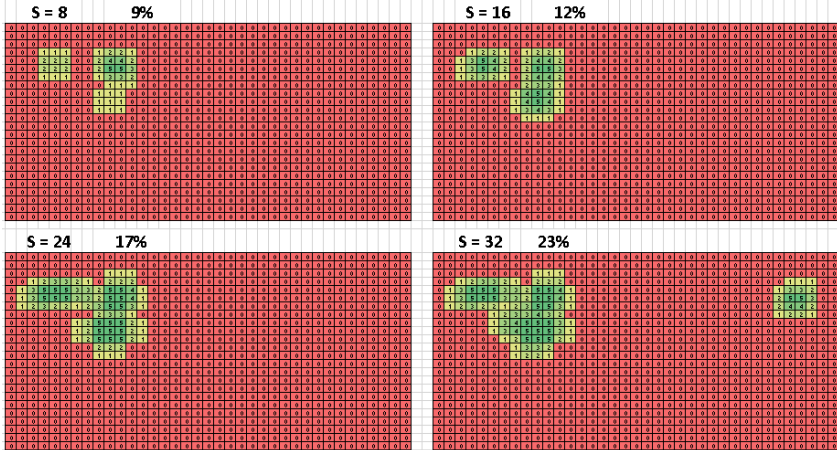


Figure 18: The weighted revised model with $l_{ij} = 5$ and $k_{ij} = 2$ for $S = 5, 10, 15,$ and 20 .

6 Summary

The ultimate goal of this pilot study is to maximize the network coverage for the desired demand with the limited supplies of equipment. Considering the advantages of installing wireless transmission equipment on street light poles in comparison to the traditional telecommunication towers, the linear model generated in this study provides the optimal locations in the City of Richmond to install cell equipment. The optimal solution generated by the model would significantly improve the network coverage when compared to the existing Wind Mobile's coverage. Furthermore, through statistical analyses, transit ridership and age are excluded in the revised model. This model is formed based on: population density, points of interest, income, and Wind Mobile telecommunication towers. Comparing results obtained from the primary model and the revised model, the latter is preferred because it is able to meet the demand while controlling coverage overlaps. The optimal locations are generally scattered across areas with high demand, i.e. City Centre and nearby neighbourhoods. Moreover, unnecessary overlaps should be avoided to provide larger coverage; on the other hand, the allocation of two telecommunication equipment near each other can improve network quality in order to satisfy high consumer demands. In general, all

of the circumstances above should be taken into consideration in the long-term decision making process.

7 Further Improvements and Expansions

While it is feasible to incorporate building heights into the existing models, the problem is modeled in the two-dimensional (2-D) space rather than the three-dimensional (3-D) space for this pilot study. In most cases, the frequency range of the equipment is significantly higher than the building height itself and short enough to not cause any radio frequency disruptions to overpassing aircrafts. Hence, modelling in 2-D while considering the building heights would imply maximal spherical radius coverage for each equipment. However, if the cell equipment with higher frequencies are placed on top of tall structures (e.g. high rises) on a very high terrain, then the consumers that reside and/or work in the lower terrains will be unable to receive any network reception. Modelling in 3-D would be a more realistic and accurate approach in future works on this topic.

According to the City of Surrey report for the pilot program [15], the space between two street lights is between 30 to 60 meters, so each side of all 888 grid cells contained between 10 to 20 street lights. In this paper, it is assumed that the cell equipment will be set up in the middle of each cell, since setting it on other street lights may slightly shift the network coverage; even though, such a minor shift can be ignored due to its minuscule impact considering the whole area (129.3 km²) of the City of Richmond. In order to create a more precise result, the study region can be divided into many more grid cells, that is, to apply a grid where each cell covers a much smaller area.

When proposing a specific location for such equipment, one needs to consider external obstacles and restrictions imposed on the project: the city hall's agency over the contract's content, and the accuracy and timeliness of the gathered data. For example, the Surrey pilot plan in 2012 restricted each telecommunication providers to install a maximum of four equipment devices. An appropriate adjustment can be made in regards to these restrictions from the municipalities in question, since their decisions can drastically affect the optimal solution. Moreover, the ideal location for installing wireless transmission equipment requires certain height and a non-blocking environment for network signal transferring.

Although for the most part, the factors considered under these models are relatively stable, additional factors such as: the existing location of antennas, the coverage radius, and the capacity of these devices are changing on regular basis due to rapid development in the mobile technology. Therefore, obtaining timely information is critical in acquiring a higher accuracy level. The models developed in this study for solving the optimal location of telecommunication equipment can be easily applied to other jurisdictions, where they are considering to implement a similar pilot study.

A Point of Interest

A.1 Kwantlen Polytechnic University

Kwantlen offers bachelor's degrees, associate degrees, diplomas, certificates and citations in more than 200 programs. More than 17,500 students annually attend Kwantlen campuses in Surrey, Richmond, Langley and Cloverdale [16].

A.2 Richmond Centre (Mall)

By the number of stores, it is the largest mall in Richmond with 250 stores and services [15].

A.3 Richmond Night Market

At its peak, the Richmond Night Market had over 400 booths selling mostly Asian cuisine, merchandise, and entertainment. Over 30,000 people visit the night market each night [17].

A.4 River Rock Casino Resort

The casino has 10,000 visitors per day on average and generated an annual revenue of \$244 million as of August 2006. In addition the resort has a hotel with 220 rooms and River Rock Show Theatre that can seat 995 Theatre style, 460 Cabaret and Banquet style [18].

A.5 Vancouver International Airport (YVR)

Canada's second busiest airport, Vancouver International Airport welcomed 17.97 million people in 2013, facilitated more than 300,000 aircraft take-offs and landings and handled over 228,000 tonnes of cargo [19]. YVR experienced its busiest day the day after the closing ceremony of 2010 Vancouver Winter Olympics where close to 40,000 passengers departed the airport [20].

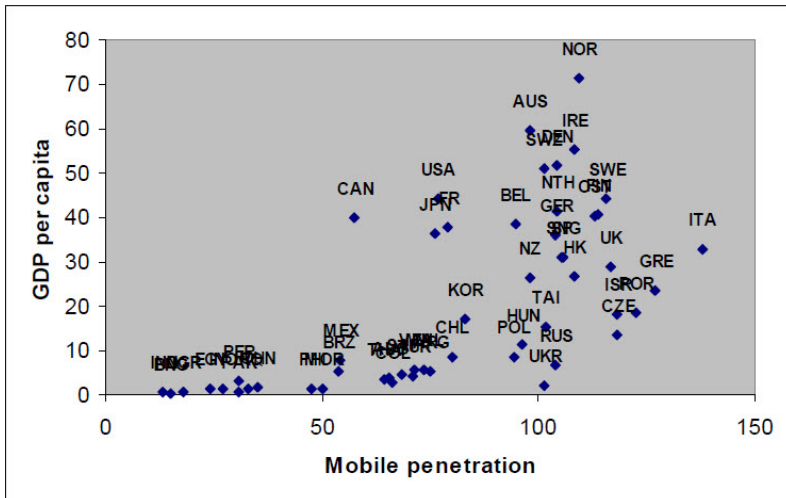
B Statistical Tables

B.1 Correlation Between Factors

Correlation Matrix							
	Populaton	Transit	Income	Age	Interest	Eij	Fij
Population	1.00	0.70	0.37	0.76	0.21	0.43	-0.43
Transit	0.70	1.00	0.56	0.78	0.26	0.42	-0.42
Income	0.37	0.56	1.00	0.54	0.21	0.32	-0.32
Age	0.76	0.78	0.54	1.00	0.18	0.42	-0.42
Interest	0.21	0.26	0.21	0.18	1.00	0.29	-0.29
Eij	0.43	0.42	0.32	0.42	0.29	1.00	-1.00
Fij	-0.43	-0.42	-0.32	-0.42	-0.29	-1.00	1.00

Table 13: Correlation between factors

B.2 Relationship between income and phone usage



Data source: Campbell and Chen, 2007; mobile subscriber units per 100 persons and GDP per capita in US\$ (000s).

Figure 4-1
Mobile Adoption and Income

Figure 19: Relationship between income and usage of cellular phones

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