The Community Mailbox Location-Routing Problem *

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Abstract

Canada Post has recently decided to phase out door-to-door delivery in urban areas in favour of building community mail boxes. This change in delivery method will require all households to walk to their nearest community mail box (CMB). Since operations costs will now largely be dependent on transportation costs, we aim to find locations for community mailboxes while also planning routes for the delivery of mail to these boxes. The purpose of the community mailbox location-routing problem is to place a minimal number of CMBs within a reasonable walking distance to each house, and find the shortest route for delivery. Our model uses a combination of heuristic methods including clustering and the sequential use of a location allocation problem and travelling salesman problem. We obtained successful results applying our model to a small neighbourhood in Delta, BC on a single test cluster.

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1 Introduction

In the community mailbox location-routing problem, our goal is to determine locations for community mailboxes from a set of potential locations and to plan delivery routes to the selected locations. A community mailbox (CMB) is a large steel box composed of smaller locked compartments assigned to each household and a small number of larger boxes for parcels and packages. A key for these boxes is placed into the package owner's assigned compartment when they receive a package. We aim to minimize travel costs and have estimated them to be proportional to the distances mail trucks will be travelling. Minimizing costs also means minimizing the number of potential locations selected for use; however it is also important to ensure that residents are not forced to walk too far to their assigned CMB. We have chosen to apply our method to a neighbourhood of 3,038 houses in Delta, BC as a case study.

The problem of simultaneously deciding location placement and route plans is known as the location-routing problem. The community mailbox location-routing problem can be specifically described as a single depot, multi-vehicle, multi-facility problem, where the facilities are CMBs. Few special cases of the location-routing problem are solvable with exact methods. In this paper we will use heuristic methods to approximately solve this problem. First, we will use a clustering based approach to divide the neighbourhood into smaller areas, each area being serviced by a single vehicle. Next, we will use a sequential heuristic to determine the facility locations through a location selection problem followed by solving a travelling salesman problem for each cluster in order to obtain the routes.

Before applying our method to real data, we have surveyed a local neighbourhood in Langley, BC which currently uses community mailboxes. This review allowed us to obtain information such as which areas are appropriate for community mailbox placement and their approximate distances from houses. We then applied our method to our chosen location with an already existing mail depot, servicing postal codes V4L and V4M. In this neighbourhood, there exists different types of residences: houses, apartments, businesses, and several rural areas with community mailboxes in use. Our model assigns each house to a community mailbox and then plans routes which will service all community mailboxes and apartment complexes.

In conclusion, we were successfully able to assign each house to a CMB for a cluster of 855 houses. Each house in the cluster was assigned to a CMB within 300 metres. The cluster we worked with originally contained two walking routes, and based on sizes of current CMB delivery route, we believe this can now be done by a single employee. The delivery route for this cluster was calculated by solving a travelling salesman problem starting from the mail depot to each selected CMB and returning to the CMB.

2 Background

Canada Post is Canada's primary postal operator and has more than 6600 post offices across the country. They offer various shipment services across Canada, from everyday household mail to large parcels. In recent years, the number of domestic mail pieces sent and received by Canadians has decreased, from 2008 to 2012, this decrease has totalled 23.6%. [2] The long term trend does not look optimistic and Canada Post believes a change in services is required in order for Canada post to remain self-sufficient.

Canada Post has investigated different initiatives to solve this problem. They claim the most cost effective solution to be the implementation of the community mailbox and the end of door-to-door delivery in urban areas. Although residents now face a walk of several minutes to receive their mail, Canada Post advertises CMBs as more secure and convenient. This is especially true for receiving parcels and packages, they will be safely locked up until they can be picked up at the residents' convenience.

Currently about two thirds of Canadians rely on methods other than door-to-door delivery. This fraction mainly includes residents living in rural areas and apartment buildings. Over the next five years, door-to-door delivery will slowly be phased out in favour of CMBs for the remaining one third of Canadians. Canada Post has already estimated a cost savings of $175 annually per address once all CMBs have been built. These cost savings are primarily due to less employee time spent delivering mail.

Now that Canada Post has decided their plan, it needs to be put into action. Locations for CMBs need to be found, and new delivery routes need to be established. According to Canada Post's "Five Point Action Plan" [2], sites are chosen according to factors such as safety, accessibility and proximity to the address they serve". Through the use of Operations Research techniques, site locations and delivery routes can be calculated to maximize the savings for Canada Post. Although there will be many other costs involved with the shift to CMBS; such as costs to construct CMBs and the possibility of needing to purchase more vehicles, concentrating on long term costs is important since they will have the highest costs over time.

3 Survey of a Neighbourhood using CMBs

CMBs are in use in many newly developed areas. Investigating one of these areas has provided us with useful information for location selection, route size, and CMB capacity. Canada Post has publicly available data on all their letter routes, including house counts and delivery methods. Finding routes which primarily deliver to CMBs allows us to estimate a feasible number of houses to assign to each delivery route. After finding such a route in the area of 96 Avenue and Glover Road in Langley, BC a survey was done using Google Street view. With their technology, we were able to browse through any street with an accurate visual of how the location looks as if we were actually there in person. We were then able to record all locations of CMBs and their respective capacities. CMB placement is a difficult task since potential locations must be put in an area that does not upset residents. A survey of a CMB neighbourhood has shown that the placement of these boxes are quite versatile. The primary location for placement is on land with no houses such as park entrances, pedestrian alleyway entrances, and on land between property fences and sidewalks. In the absence of areas such as these, CMBs have been placed on land on the opposite side of a sidewalk from houses. Also from these routes, distances from houses to their assigned CMB are measurable. This information helps us estimate and upper bound for the distance residents must walk.

In a sample of 33 suburban service mail routes in Surrey, BC, the average number of houses on each route was 806 houses (minimum 566, and maximum 1,018). Suburban service mail routes are defined as “Delivery by contractor to group mailboxes usually near or on the perimeters of urban areas" [4]. These routes include both community mailboxes and delivery to rural areas. Since these large routes are similar in nature we have used this average as a guideline to formulate the size of our delivery routes. This will give us a close estimate for both the capacity of the mail trucks as well as the time required for delivery.

The maximum house to CMB distance that we recorded in this area was approximately 250 metres. In our problem we have chosen to place an upper bound of 300 metres. At an average walking speed of 5 km/hour this length would take 3.6 minutes to walk while 250 metres would take 3 minutes to walk. Most houses are less than 3.6 minute walk while there exists some houses on this upper bound.

The CMB capacities we have seen on this route have ranged from 33 to 72 houses. In our problem we have chosen to add the constraints of a minimum of 5 houses which would fill the majority of the smallest 9 house CMB and a maximum of 60. We have lowered our number from 72 because the neighbourhod in Langley was newly developed and designed to have the CMBs in place. The large 72 house CMB takes up a lot of room, and may be too large for the potential locations in Delta.

4 Model Description

In our model, multiple mail trucks will make tours starting from the depot and returning back to the depot once all CMBs on the route have been serviced. The size of large scale problems such as this one causes an increased difficulty. Therefore, our first step will be to cluster the neighbourhood area into smaller areas. This simplifies the problem by reducing the number of customers to solve for and puts it into a well-studied form. If we assign a single vehicle to each cluster then we can then solve a single depot, single vehicle, multi-facility problem. Each cluster will use the same depot, and will have approximately the same number of houses.

Given the size and nature of this problem, we have investigated several heuristic methods for formulating a model. In the paper "Location-routing: issues, models and methods" [5], the authors describe the sequential method heuristic as "first solving the locational problem by minimising the sum of depot-to-customer distance and then solving the routing problem based on the depot locations found". In our problem the facility-to-customer distances will always be very small compared to most studied problems. We do not believe that a difference in walking distance of several hundred metres to significantly impact residents. Thus, we have placed an upper bound on the distance residents may walk to their CMBs and have chosen to minimize the number of CMBs instead. We believe this to be a much more cost saving method, as it will lead to less stops and consequently less time taken to deliver mail.

In the same paper, the authors state that clustering-based methods are similar to the sequential method since there isn't any feedback between location selection and the routes. They do however claim that using this method improves location and routing decisions when the "clustering is based on some skeleton of a routing plan". Since we are basing our clusters on pre-existing mail delivery routes as our "skeleton", we have confidence in our choice of model.

From our survey of the neighbourhood with CMBs in use, we have seen that CMBs can have many different capacities. The smallest size CMB has capacity for 9 houses, from this we make a reasonable assumption that a minimum 5 houses need to be assigned to each CMB, since CMBs do not need to be full. The largest size we have seen contained enough compartments for 79 houses. Since there are different sizes and styles of CMBs we have set the maximum capacity of CMBs to be 60 houses. This maximum can be increased if desired and if there exists a potential location with large enough area to hold a large volume CMB.

Another assumption we have made is to assume all apartments will have their own CMB inside their building if they do not already. Businesses will also be treated the same as houses. When dealing with houses in the rural areas of the neighbourhood, they will not be subject to the same walking distance upper bound. This is because many of these residences are located very far away from each other, and it is impossible to subject them to the same constraints. A good solution to this problem is to place their CMBs on common streets that they will pass when going to or from their houses.

5 Model Formulation - Location Selection

Notation and Variables

i: the set of potential CMBs in each cluster k, $i = \{1, 2, 3, ..., n\}$
j: the set of houses in each cluster k, $j = \{1, 2, 3, ..., m\}$
k: the set of clusters, $k = \{1, 2, 3, ..., p\}$

$$y_i = \begin{cases} 1 & \text{if CMB } i \text{ is used as a facility} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if CMB } i \text{ is assigned to house } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ij} = \begin{cases} 1 & \text{if CMB } i \text{ is a possible assignment to house } j \\ 0 & \text{otherwise} \end{cases}$$

Model

$$\text{min} \sum_{i=1}^{n} y_i \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{m} x_{ij} * z_{ij} \leq 60 \quad \forall j \in k \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} * z_{ij} \geq 5 \quad \forall j \in k \tag{3}$$

$$\sum_{j=1}^{n} x_{ij} * z_{ij} = 1 \quad \forall i \in k \tag{4}$$

$$x_{ij} - y_i * 60 \leq 0 \;\; \forall i, j \in k \tag{5}$$

$$x_{ij}, y_i, z_{ij} \geq 0 \;\; \forall i, j \in k \tag{6}$$

To solve the location selection problem in Excel, we first input data for each z_{ij} which are the binary variables representing whether house j is within 300 metres of CMB i. This data is collected using iMapBC, a mapping software found online which easily allows us to find the distance between two different points. For each potential CMB, we measure street distances of 300 metres and input $z_{ij} = 1$ for all houses that are within this distance. The binary decision variables x_{ij} are then selected in such a way that minimises the sum of y_is shown by equation 1 in the mathematical model. Equations 2 and 3 correspond to the constraints for allowable numbers of houses to be assigned to each CMB. There is a maximum of 60 houses and a minimum of 5 houses. Equation 4 ensures that only one potential CMB is chosen for each house. Finally, equation 6 is a linking constraint which ensures that houses can only be assigned to CMBs which have been selected.

This will be negative when $y_i = 1$ and equal to zero when $y_i = 0$ which is acceptable for $x_{ij} = 1$, if $x_{ij} = 1$ and y_i then it is not a feasible assignment, this would assign a house to a CMB that has not been selected.

6 Formulation - Travelling Salesman Problem

Notation and Variables

i: the set of CMBs in each cluster, $i = \{1, 2, 3, ..., n\}$
j: the set of CMBs in each cluster, $j = \{1, 2, 3, ..., m\}$
k: the set of clusters, $k = \{1, 2, 3, ..., p\}$
c_{ij}: distance between CMB i and CMB j

$$x_{ij} = \begin{cases} 1 & \text{if arc } x_{ij} \text{ is chosen for the route in cluster } k \\ 0 & \text{otherwise} \end{cases}$$

Model

$$\min \sum_{i=1}^{n} c_{ij} x_{ij} \tag{7}$$

$$\text{s.t.} \sum_{i=1}^{m} x_{ij} = 1 \quad \forall j \in k \tag{8}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \in k \tag{9}$$

$$\sum_{(i,j) \in S} x_{ij} \leq |S| - 1, \quad \forall S \in V, \quad |S| > 1 \tag{10}$$

The travelling salesman problem (TSP) considers the problem of starting at one location (the depot), travelling to multiple cities (CMBs), and returning back to the location using the shortest cycle possible. This problem has been extensively studied and has a variety of methods to solve it. Using the model suggested in “Optimization Models with Spreadsheets" [1], we are able to formulate a model for this problem on Excel.The formulation consists of two parts: an array of shortest distances from each CMB i to every other CMB j (when $i = j$, $c_{ij} = 999$ to remove any chance of this infeasible path being chosen), and an array of decisions variables which determines the travel order. There must be exactly one path to every CMB and exactly one path from every CMB. Equations 7 and 8 ensure that the sums of both the rows and columns of the travel order array sum to one in order to satisfy this constraint. These constraints are not sufficient to always achieve a feasible solution to the TSP which necessitates the use of subtour elimination constraints. Without subtour elimination constraints we may obtain a solution that contains subtours. Subtours occur when the optimal solution does not consist of a single visit to each city, but starts at and returns to more than one city for multiple cities. Adding in subtour elimination constraints is done individually and is different for each problem. This method is not practical for large size problem because the number of constraints increases at an exponential rate. The exact number of constraints is calculated by $2^{n-1} - n - 1$. In a problem with 14 cities such as ours there is the possibility for over 8000 subtours to occur. Adding these constraints individually consists of finding the optimal solution, checking for subtours, adding a constraint to prohibit a particular subtour if it occurs, and repeating until a solution with not subtours is obtained. The subtour elimination constraint requires that the sum of the pairs of CMBS in the subtour be less than the number of cities in the subtour. Minimising the total distance of the tour is done by the objective function, Equation 6 which minimises the sum of chosen arcs used in the tour.

7 Input Data - Residences and Clusters

Using information on how many houses are on each route as well as the locations of each route, we are able to combine selected routes together to form single clusters. The criteria we used to cluster the routes together was proximity to each other and the number of houses. Since our selected area is quite small and has only 11 door-to-door walking routes, we are able to decide on the clusters through trial and error. We tested possible clusters by grouping adjacent routes together until we found a suitable combination. We used the average of 806 houses previously calculated as a rough guideline for this, and have chosen the clusters as follows:

- Cluster 1: LC0020, LC0021, LC0022 (total 927 houses)
- Cluster 2: LC0023, LC0024 (total 855 houses)
- Cluster 3: LC0013, LC0026, LC0027, LC0028 (total 845 houses)
- Cluster 4: LC0008, LC0025 (total 679 houses)

We have chosen cluster 2 as our sample cluster, Table 1 shows the data on quantity and types of residences. We assume that apartments have an area similar to a CMB in the building. Businesses will be treated the same as houses. We assume that none of these businesses continuously receive large parcels that require a different setup. Figure 1 shows a mapped area of the neighbourhood we are focusing on, with our selected cluster highlighted.

Route	Houses	Apartments	Farms	Businesses	Total
LC 0023	400	0	0	1	401
LC 0024	371	57	0	26	454
Total	771	57	0	27	855

Table 1: Number and types of residences in our sample cluster, will be referred to as houses for simplicity.

Using this data for our case study, the previously mentioned variables will become:

- The number of potential CMBs, $n = 37$

- The number of houses, $m = 855$
- The number of clusters, $k = 4$

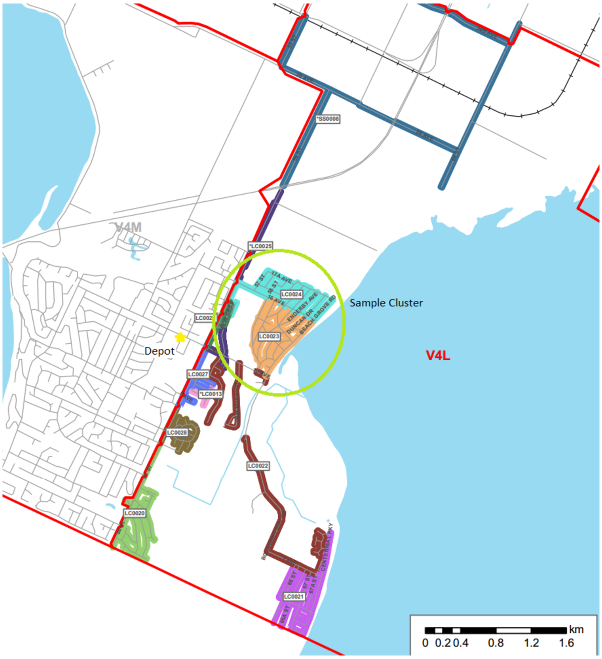

Figure 1: Map of current letter carrier walks.

8 Input Data - Potential CMB Locations

Potential CMB locations were chosen with the use of Google Street view. We kept record of all possible locations on a separate map. Acceptable locations for CMBs for our study were found fairly easily in the same type of locations seen in other neighbourhoods (park entrances, pedestrian alleyway entrances, land between property fences and sidewalks, and land on the opposite side of a sidewalk from houses). In total we found 37 potential CMB locations in our sample cluster as shown in Figure 2. The closest CMBs within 300 metres of any house are kept track of as potential CMBs to be assigned to these houses ($z_{ij} = 1$).

9 Outputs and Results

Both the location selection and travelling salesman problem were solved using the Open Solver add-on for Excel. Results for the location selection problem were obtained in less than a

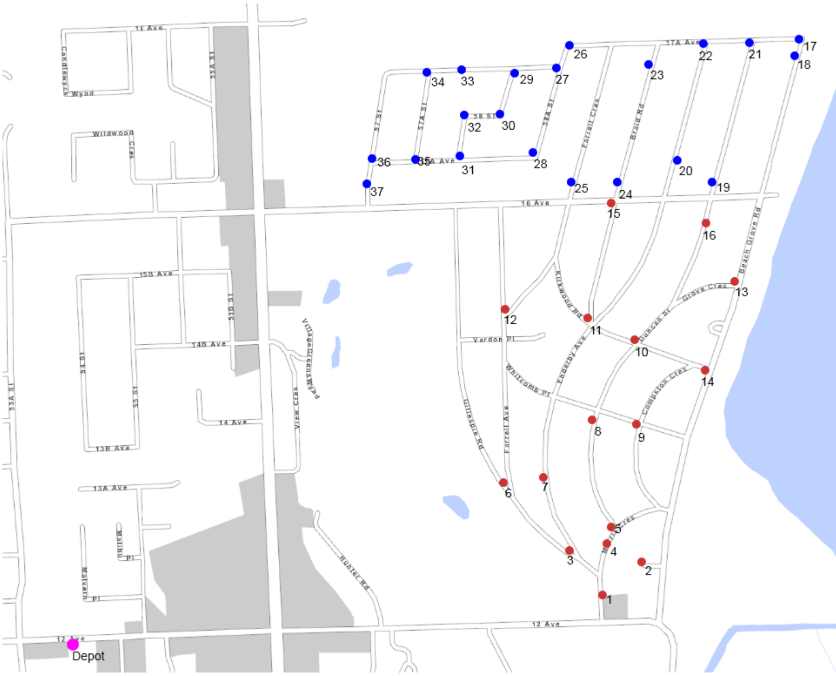

Figure 2: Potential CMB locations.

minute, a very short amount of computational time. The TSP required several seconds of computation time to solve, but required extra time to manually add each subtour elimination constraint. Figure 3 shows an illustration of the results we obtained through the location selection problem. The number of CMBs was minimised to a selected 14: 3, 5, 8, 11, 12, 13, 15, 17, 21, 22, 25, 29, 35, 37. They were then relabelled from 1 to 14, Table 2 shows the capacities of these selected CMBs.

CMB #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Capacity	53	57	60	43	60	60	58	43	46	60	60	55	34	22

Table 2: Capacities of selected CMBs

After obtaining CMB locations, we solve the TSP starting at the depot, visiting each CMB and returning back to the depot. The results we obtained are illustrated in Figure 4. This route is the final output of our model. It demonstrates the most cost effective method of delivery while also ensuring residents have a reasonable distance to walk to pick up their mail.

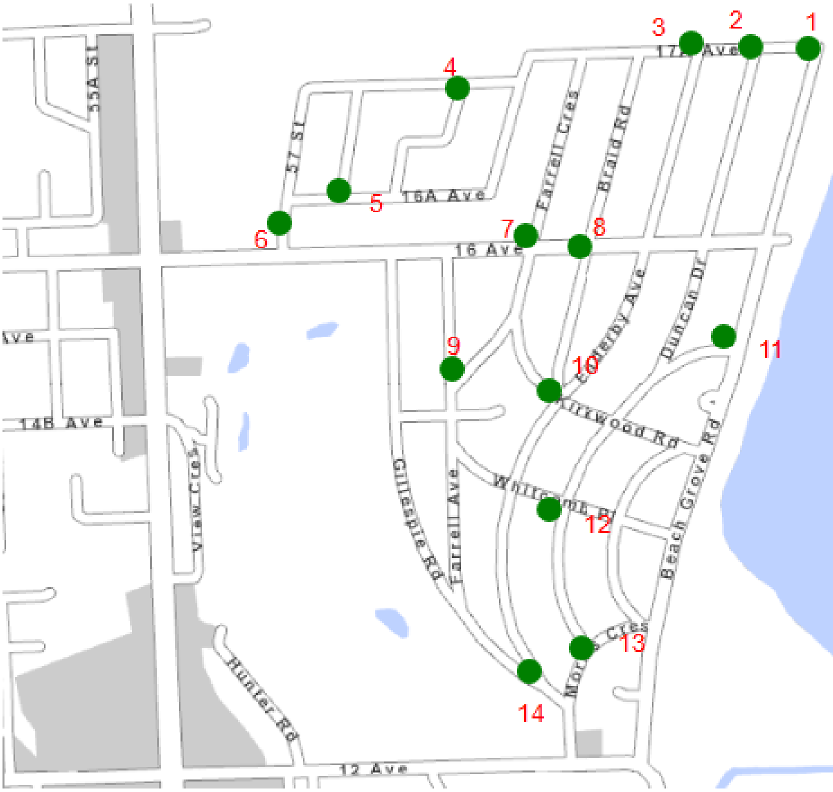

Figure 3: Results obtained from solving location selection problem

10 Conclusion and Future Developments

In conclusion, we were able to model a small neighbourhood in Delta, BC into both a location selection problem, and a travelling salesman problem in order to determine optimal locations for CMBs. We finished our model by planning the routes to delivery to the selected CMBs. The two routes used in this cluster are currently delivered door-to-door by two Canada Post employees, using the driving route and CMB placements suggested in this paper, these two routes can now be serviced by a single employee. This saves time and money for Canada Post. In the area we studied, cluster 3 was made up of four door-to-door delivery routes. Combining them into a single route delivering to CMBs would reduce four employees down to one, even further reducing costs.

In future models, the size of the results can be scaled up to service all door-to-door routes currently made by this mail depot. More sophisticated software would allow for this development to occur with much more ease. An improved method for collecting data on distances as well as more specialized software for solving travelling salesman problems would allow for more scalability and more accurate results.

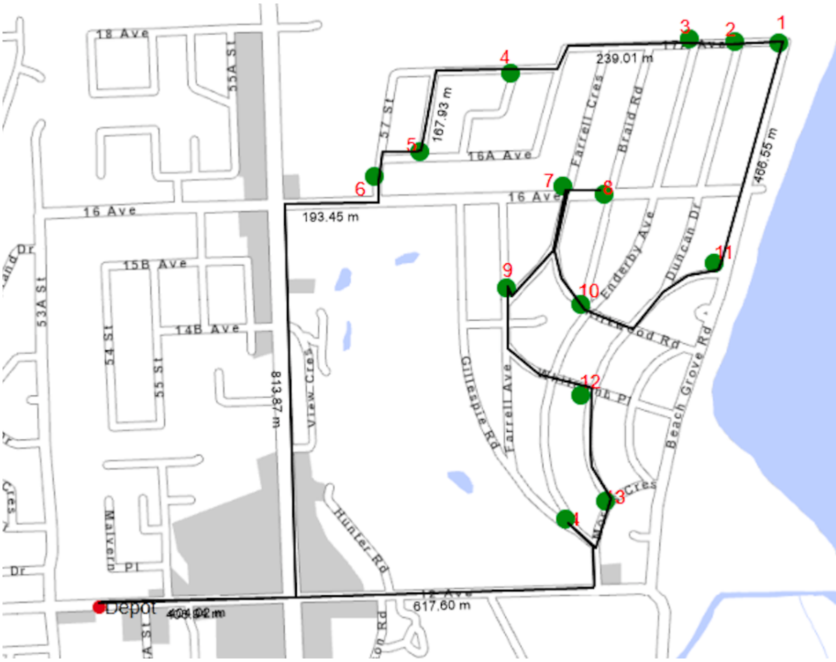

Figure 4: Results obtained from solving travelling salesman problem

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