Optimization Model for Network Coverage of SFU Bennett Library

Kaveh Alemi, Anson Christo, Angus Hon, David Ling

Department of Mathematics, Simon Fraser University,
Surrey, BC, Canada

Abstract

In this paper, we consider the problem of optimal WiFi router location for the case of Simon Fraser University's Bennett Library. We have made it our goal to implement a mathematical model that takes into consideration the level of demand for WIFI on every floor of the library and allocates routers on each respective floor appropriately. Every library floor will be split into 40 sections where each contains a standardized index score between 1 and 5, which represents the demand level of that section. Demand scores will be derived from the amount of space a section has for individuals to be stationary and connected to the WIFI. In order to quantify this value for each section, we will count the number of chairs, tables, couches, and wall-plugs a section contains. Considering these demand scores for every floor, an appropriate allocation of routers can be made in an effort to maximize demand coverage. Moreover, users may set an upper or lower bound for the number of routers that are dispersed on a given floor. All intermediate steps, taken in the construction of this mathematical model, are further explained in the Model Construction section of this paper. Furthermore, our model has been only applied to floors 2, 3, 4, 5, 6 out of the seven existing floors of the library since floors 1 and 7 do not have adequate study spaces. Our results have been subjected to a certain upper-bound of commercial-grade routers, and the allocation of these routers followed a fairly common pattern through all of the floors. All routers were allocated to demand concentrated sections in order to ensure that coverage is maximized. Further insight and information are provided in the Results section of this paper.

1 Introduction

Simon Fraser University's Bennett library offers many top notch facilities to students such as computing labs, study areas, and study rooms but unfortunately lacks in the area of WIFI. The library's WIFI system is reputed to have a variety of issues such as weak signal strength, lack of coverage, and constant disconnections. As students of this university and avid visitors of this facility, we have taken the initiative to improve the current WIFI system with the help of our Operations Research skills. Our initiative begins with considering the optimal location of WIFI routers that result in maximum user coverage on a given floor of the library. The aforementioned variables, router location, and WIFI coverage will be the variables of interest and the objective, respectively. Through constructing a mathematical model that maximizes the objective by assigning appropriate values for the variables of interest while taking into account capacity and proximity constraints, a potentially improved router floor-plan can be derived. It should be noted that for our solution to be more plausible, the economic effects of the number of routers allocated must also be taken into consideration. As a result, this model should allow users to control the number of routers that can be allocated within a single floor. This will allow for the results of this model to align more with the user's budget for WIFI. If we are able to propose an improved floor plan for the WIFI system, then stakeholders such as students, faculty members, and facility users will be able to receive an adequate WIFI signal with minimal disconnections. Furthermore, this paper delves deeper into the construction of the mentioned mathematical model and also provides an analysis of the obtained results and how they compare with the current state of the library's WIFI system. Sections that detail relevant assumptions and limitations are also included at the end.

2 Model Construction

In this section, we will be discussing how the proposed model was created, and then later implemented for each floor of the library [**? ?**]. The first step in the construction is to split each floor into a grid of 40 square sections. The reasoning for 40 equal squares is because each floor of the library consists of an area with an approximate length of 100 meters and width of 40 meters. The following diagram shows the third floor of the library split into equal sized sections.

We will then allocate standardized demand scores to each section. These scores will correspond to the demand for WIFI for each section on every floor. A key factor in working with WIFI is realizing that the allocated routers will provide WIFI for a surrounding diameter of 30 meters [**?**]. Therefore, to incorporate this overflow of WIFI from one section to the other, we have chosen to consider a set of adjacent sections that an allocated router will provide WIFI for. To make this overflow of WIFI consistent with the area of each section, we are including sections to the left, right, bottom, top, top-left, top-right, bottom-left, and bottom-right of any given section to be included in its adjacent

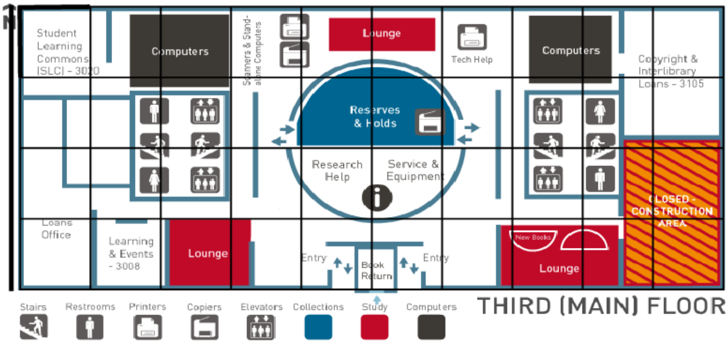


Figure 1: **Example of the sectioning of a floor**

set. There are obvious exceptions, where adjacent sections fall outside of the 10 by 4 floor plan, and hence must not be included in the set. Furthermore, the objective for each model of every floor will be the total demand scores covered, provided by every allocated section and its respective adjacent set. Further intermediate steps and calculations are provided below.

2.1 Parameters

In this section we will discuss the parameters and any mathematical notations used in this model. As previously mentioned, in order to consider the demand for WIFI for different locations of each floor, we must assign a demand score, D_i, for every section.

D_i : Demand score reflecting WIFI demand for a given section i, with values in the range [1, 5]. Note: We have not specified which floor, section i is being mapped in since this model is independently implemented for each floor of the library.

When D_i is given a value of 1, this entails that the section i has very low demand for WIFI. Conversely, a value of 5 corresponds with a very high demand for WIFI. Moreover, how each score is derived for each section is further discussed in the Calculations sections.

In order to model the overflow of WIFI from one section to the other, we previously stated that we are considering an adjacent set for each section. Moreover, each set will contain all sections in a 30 meter diameter of any given section i. To model this set in a linear program, we must consider a binary variable that expresses the adjacency of sections i and j. By introducing the binary variable $A_{i,j}$, we can factor in the overflow of WIFI from section i to section j.

$$A_{i,j} = \left\{ \begin{array}{ll} 1, & \text{Section i and section j are adjacent within a 30m diameter} \\ & \\ 0, & \text{Otherwise} \end{array} \right\}$$

There following diagram is an example of any arbitrary section i and what is considered an adjacent section j.

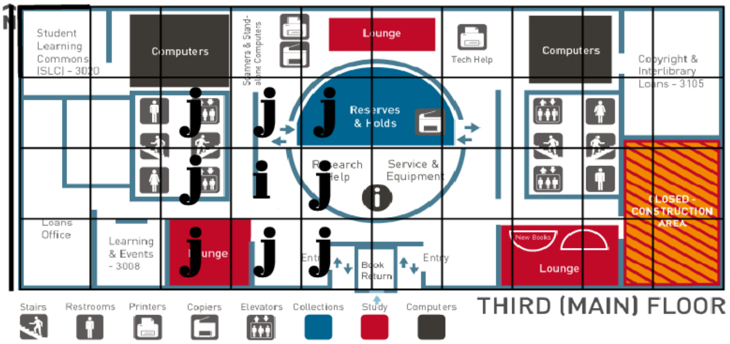


Figure 2: **Example of an arbitrary section i and corresponding adjacent sections j**

2.2 Decision Variables

As stated in the introduction, the variables of interest, or decision variables, of this model are the sections for which a router is allocated to. By utilizing a binary variable for every section, we can easily determine if a router is allocated to any given section i. More specifically, the binary variable X_i will represent the status of a router being allocated to section i. There will be a total of 40 decision variables for every floor's model.

$$X_i = \left\{ \begin{array}{ll} 1, & \text{Router Allocated to Section i} \\ & \\ 0, & \text{Otherwise} \end{array} \right\}$$

2.3 Objective Function

The goal or objective function of this model is to maximize the total satisfied demand for WIFI across each floor plan's 40 sections. More specifically, by considering the demand scores of both the sections for which a router is allocated to and all respective adjacent sections, the total demand coverage can be derived. By maximizing this value, we can determine the optimal locations for which routers can be allocated to. Moreover, to model this as an integer linear program we must refer back to the parameters mentioned earlier. By computing the sum-product of the following variable, X_i, the total demand coverage can be derived. Therefore, the objective function of this model, for which we are maximizing, will be the following.

$$\text{Max} \sum_{j=1}^{40} \sum_{i=1}^{40} A_{ij} D_i X_i$$

Which further breaks down to...

$$\begin{aligned}\text{Maximize } F(X) = &\sum_{i}^{40} 9D_iX_i \text{ for inner nodes}\\ &+\sum_{i}^{40} 6D_iX_i \text{ for edge nodes}\\ &+\sum_{i}^{40} 4D_iX_i \text{ for corner nodes}\end{aligned}$$

The function indicates there are 9 adjacent sections for inner nodes, 6 adjacent sections for edge nodes and 4 adjacent sections for corner nodes.

2.4 Constraints

For this model there are two main categories of constraints: Capacity Constraints, and Proximity Constraints. They are further explained below.

Capacity Constraints: In order to make this model more realistic and adjustable for the user's needs, we must make the already derived model more considerate of economic factors. As a result, a constraint has been included which allows for an upper-bound to be set for the number of routers that can be allocated. In doing so, an improved result can be derived that matches the user's budget for WIFI for every floor. The following equation can be applied for every model that corresponds with any of the library floors:

$$\sum_{i=1}^{40} X_i \leq \text{Capacity} \quad \text{[Any Floor]}$$

Since we have obtained information about the number of routers on every floor of the library with the existing WIFI system [**?**], we will use these figures as the Capacities for this category of constraints.

$$\sum_{i=1}^{40} X_i \leq 6 \quad [2^{nd}\ Floor]$$

$$\sum_{i=1}^{40} X_i \leq 7 \quad [3^{rd}\ Floor]$$

$$\sum_{i=1}^{40} X_i \leq 6 \quad [4^{th}\ Floor]$$

$$\sum_{i=1}^{40} X_i \leq 11 \quad [5^{th}\ Floor]$$

$$\sum_{i=1}^{40} X_i \leq 6 \quad [6^{th}\ Floor]$$

Proximity Constraints: To ensure that routers are not concentrated in a single section of very high demand, a proximity constraint has been added. This constraint prevents adjacent sections to have routers allocated to them. As previously mentioned, the definition of adjacency still holds. If a floor plan has an area with high demand for WIFI, this constraint prevents the model from allocating routers to adjoining areas. This allows for other areas of the floor to also be considered when routers are dispersed. To model this in a linear program, every adjacent set of the decision variables have to be less than or equal to 1. This ensures that adjacent sets do not have more than 1 router. The following equations express examples of this constraint:

$$\sum_{i=1,2,11,12} X_i \leq 1$$

$$\sum_{i=1,2,3,11,12,13} X_i \leq 1$$

$$\sum_{i=2,3,4,12,13,14} X_i \leq 1$$

$$\vdots$$

$$\sum_{i=29,30,39,40} X_i \leq 1$$

3 Implementation

Similar to other linear programs, our model is run with the Solver optimization software in Excel. Each of the five spreadsheets used to model each floor contains one Capacity Constraint, 40 Proximity Constraints and one Total Capacity Constraint. All other parameters and variables are as mentioned in the Model Construction section.

3.1 Assumptions

Prior to conducting the calculations, there are several assumptions and simplifications are necessary to reduce certain complexities. We address the following assumptions: radiation patterns of the routers, the dimensional space in which our model is held, the strength and decay of radio waves, and their interaction with objects.

The router under inspection [**?**] has a specific azimuth radiation pattern that represents its precise area and angle of coverage. This would suggest modelling in three dimensions to precisely represent the coverage area. Our first simplification is introduced where we ignore the Z-axis or the azimuth geometry and implement our model in two dimensions. We therefore have a simplified 2-dimensional area in the form of a circle to represent the radiation patterns of our routers.

Given that the model is represented in 2-dimensions, we make our second assumption and state that all routers are to be placed at an optimal height such that a router's height has a negligible impact on coverage and can be omitted from calculations. Therefore, our model only operates in the XY plane.

Radio waves of certain frequencies contain certain characteristics in the decay of signal strength per distance. To avoid complex mathematical calculations, our third assumption will assume the linear decay of signal strength per 10 meters (the length of a section). In addition, we will also be ignoring the interactions of radio waves with common objects such as furniture or walls within a library floor. Namely, our fourth assumption will assume that the interaction of radio waves with objects within the library are negligible.

Summary of Assumptions

- WIFI radiation pattern: two dimensional circle
- Z-axis ignored: assuming router height placement has negligible impact on coverage
- Radio waves have a linear decay in signal strength
- Negligible interactions of radio waves with objects

3.2 Calculations

In order to determine the amount of WIFI usage required in a given section, we first have to explore the amount of data required per individual user. Since we were unable to obtain usage data through SFU, we extrapolated data through alternative means. We assume that the maximum bandwidth used by a user is relative to streaming the highest quality video possible at their device's screen resolution [**?**]. If we obtain the distribution of screen resolutions over a large sample size, we can use their percentile distributions as an approximate probability of user screen resolutions at the library. Given this probability, we can compute the average Megabits per second required by a user to stream at a certain quality [Table 1].

We use unit conversion to convert values in 3 steps: total pixels to Megabytes per frame, Megabytes per frame to Megabits per frame, Megabits at 60 frames per second (required frame rate to stream videos).

$$\text{Total Pixels per frame} * \frac{1 \text{ Megabyte}}{1024 * 1024 \text{ Pixels}} \text{ per frame} * \frac{1 \text{ Megabit}}{8 \text{ Megabyte}} \text{ per frame} * 60 \text{ frames per second}$$

All values in Table 1 are calculated in this manner. For example, to calculate the Megabits required to stream a 60 fps video for the common display of 1920 pixels by 1080 pixels, the following calculations are conducted:

$$\begin{aligned} \text{Megabits for 1920 x 1080 @ 60fps} &= 1920 * 1080 = 2073600 \text{ pixels} \\ &= \frac{2073600}{1024 * 1024} = 1.9775 \text{ Megabytes} \\ &= \frac{1.9775}{8} = 0.2472 \text{ Megabits} \\ &= 14.8315 \text{ Megabits} \end{aligned}$$

With the calculated Megabits per second required per screen resolution, we can then multiply them with their respective percentiles to obtain a normalized contribution to the distribution of screen resolution's Megabits per second.

Data Usage for 60fps Video @ Given Resolution

Screen Resolution	Percentage of Users	Total Pixels	Megabits per Second	Mb/s by Percentile
1024 x 768	0.60	786,432	5.625000	0.03375000
1280 x 720	0.39	921,600	6.591797	0.02570801
1280 x 800	0.77	1,024,000	7.324219	0.05639648
1360 x 768	1.78	1,044,480	7.470703	0.13297852
1366 x 768	13.02	1,049,088	7.503662	0.97697681
1440 x 900	3.55	1,296,000	9.269714	0.32907486
1280 x 1024	1.98	1,310,720	9.375000	0.18562500
1536 x 864	0.24	1,327,104	9.492188	0.02278125
1600 x 900	3.36	1,440,000	10.299683	0.34606934
1680 x 1050	2.51	1,764,000	12.617111	0.31668949
1920 x 1080	62.07	2,073,600	14.831543	9.20593872
1920 x 1200	0.88	2,304,000	16.479492	0.14501953
2560 x 1080	0.96	2,764,800	19.775391	0.18984375
2560 x 1440	4.24	3,686,400	26.367188	1.11796875
3440 x 1440	0.49	4,953,600	35.430908	0.17361145
3840 x 2160	1.46	8,294,400	59.326172	0.86616211
other	1.68	-	-	-

Source: https://store.steampowered.com/hwsurvey/ as of Feb. 2019

Table 1: Data Usage for Given Resolutions

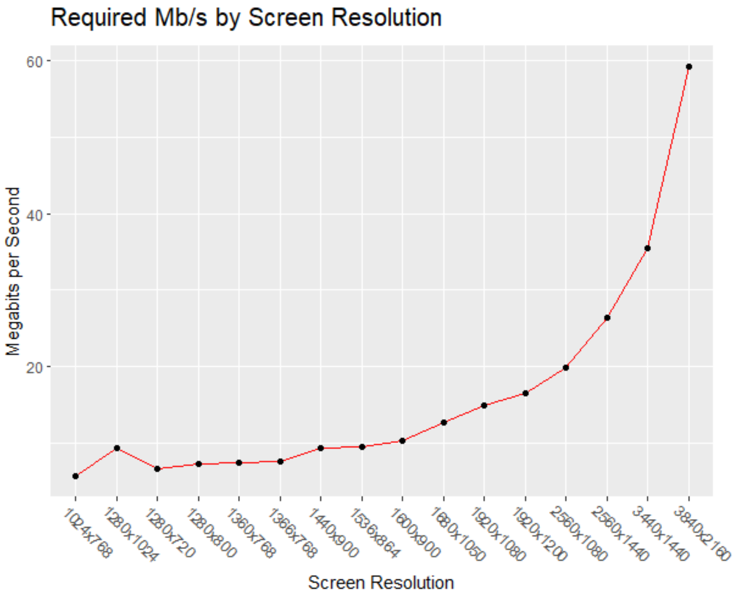


Figure 3: **Required Mbs by Screen Resolution**

From table 2, we then proceed to sum the total Megabits per Second column and divide it by the number of screen resolutions in order to obtain the average total data usage required for a single individual. The corresponding value is 14.117 Mbs. We then multiply this value by the total number of seats in each floor to obtain the minimum data usage required [Table 2].

Floor Usage Data		
Floor	Total Seats	Minimum Usage (Mbps)
2	423	5971.703
3	345	4872.985
4	194	2740.171
5	191	2697.797
6	295	4166.755

Table 2: Floor Usage Data

Now that we have obtained the minimum data user per floor, we represent the maximum data usage as double of the minimum. We then scale this value to 5 indices representing the five standardized demand scores [Table 3].

Lower Bound Index of Representative Data Usage Indices per Floor (Mbps)					
Floor	1	2	3	4	5
2	5974.703	7468.379	8962.055	10455.731	11949.407
3	4872.985	6091.231	7309.477	8527.723	9745.970
4	2740.171	3425.214	4110.257	4795.300	5480.342
5	2697.797	3372.246	4046.695	4721.144	5395.595
6	4166.755	5208.444	6250.133	7291.822	8333.510

Table 3: Data Usage Indices

The above ranges are represented in the following graph:

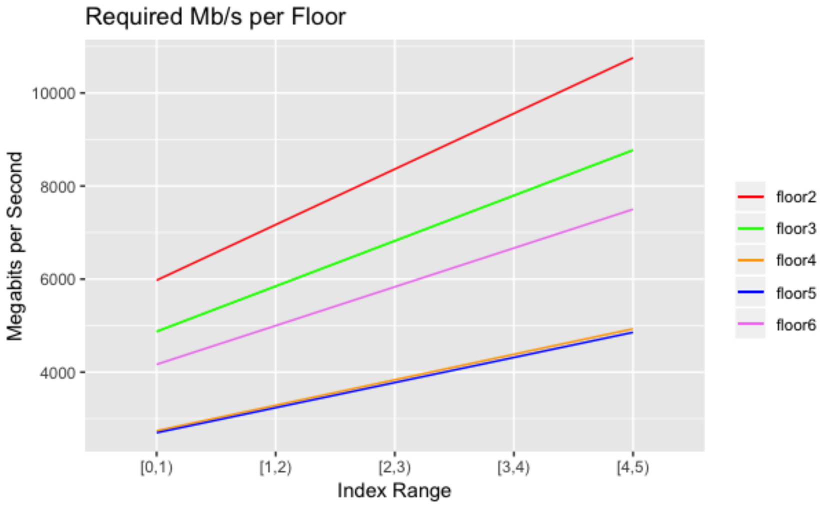


Figure 4: **Data Usage Ranges for Each floor**

The index scores calculated above represent the demand per floor. To achieve a demand per section we divide those ranges by the number of sections. In our model, each floor is split into 40 sections of equal areas. The updated index table after accounting for sections is seen in Table 4.

Lower Bound Index of Representative Data Usage Indices per Section (Mbps)					
Floor	1	2	3	4	5
2	149.368	186.710	224.051	261.393	298.735
3	121.825	152.281	182.737	213.193	243.649
4	68.504	85.630	102.756	119.883	137.009
5	67.445	84.306	101.167	118.029	134.890
6	104.169	130.211	156.253	182.296	208.338

Table 4: Data Usage Indices

Now that the standardized demand scores have been derived for each floor, to allocate these scores to the different sections we create categories. These categories all have a corresponding demand score. These demand scores have been determined by multiplying the number of chairs and wall-plugs in each section by the average Megabits usage per person (14.117 Mbs). This value then falls within an interval of the demand scores.

Demand Score Index	
Floor Layout Areas	Index Score
Study	5
Computer Lab	5
Lounge	4
General Rooms (Meeting, offices, etc)	3
Bookshelves	2
Stairwells	1
Washroom	1

Table 5: Demand Index Score

4 Results

Since our model runs for each floor individually, we have created five models to represent each floor of the library. Each floor model is equivalent apart from the demand scores of each section. The diagram in the following page shows the newly optimized router locations and the existing router layout for each floor.

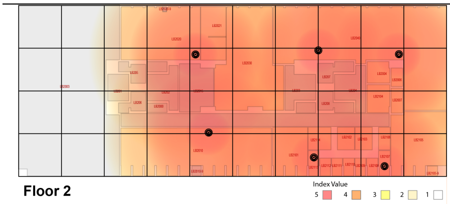
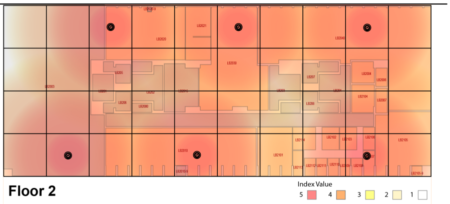
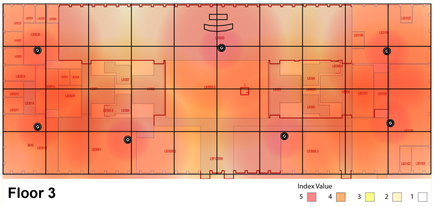
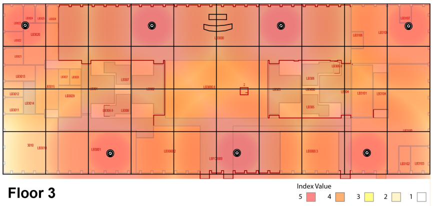
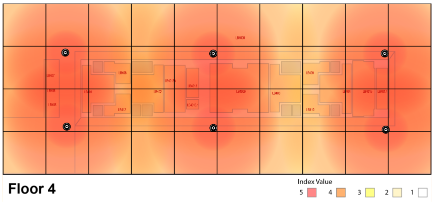
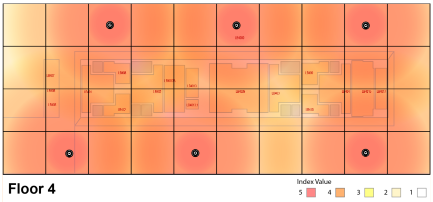
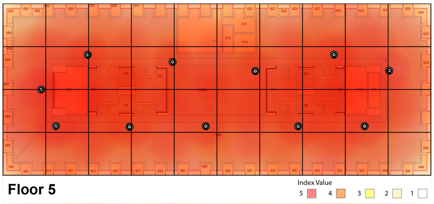
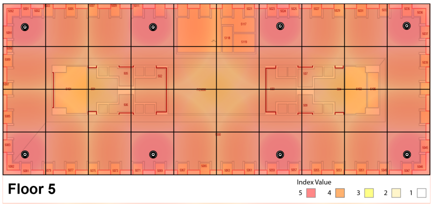
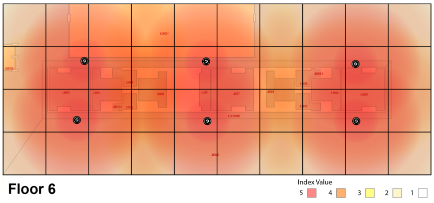
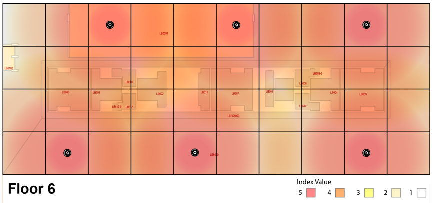
Comparison Between Old and Newly Optimized Router Layout	
Current Layout	New Optimized Layout
 **Floor 2** 	 **Floor 2**
 **Floor 3** 	 **Floor 3**
 **Floor 4** 	 **Floor 4**
 **Floor 5** 	 **Floor 5**
 **Floor 6** 	 **Floor 6**

Table 6: Comparison Map of Router Layout

The derived results express the following features:

- Floor 2 Capacity Constraint is Binding
- Floor 3 Capacity Constraint is Binding
- Floor 4 Capacity Constraint is Binding
- Floor 5 Capacity Constraint is not Binding
- Floor 6 Capacity Constraint is Binding

The optimized layouts all have a common pattern in comparison to their non-optimized counterparts, the routers are distributed more towards the outside walls of the floors. Furthermore, floor 5 poses a non-binding capacity constraint, which can result in a decrease in the budget for the overall WIFI of the library.

5 Discussion

5.1 Limitations

The limitations of this model are reflected by the lack of data in two particular categories: WIFI usage and cost for the routers. Our group was rejected when we inquired from SFU Tech Services about the demand and MB/s usage of the existing routers located in the library. This resulted in our group finding an alternative method to measure demand and hence the creation of the standardized demand scores. Additionally, when inquiring for the cost of WIFI strictly within the library, we were told this cost is not recorded separately but is rather accumulated to the SFU utility costs. The proposed model is still fully functional without these mentioned categories of data. If they were accessible, they could potentially be incorporated to make the model more accurate.

5.2 Future Iterations and Potential Improvements

We have concluded that several improvements can be done to the proposed model in order to increase accuracy and efficiency. Firstly, our proposed model consists of 40 sections because the floor plan of the library can be split into 40 equal squares resulting in a simplified map. If more sections were to be considered, a more accurate representation of demand will be introduced into the model, since there are more locations for which a router can be allocated. Secondly, in order to avoid complexity our model considers WIFI radiation patterns to be in a 2-dimensional 30-meter diameter circle. This can be modeled in a much more precise manner if the 3-dimensional radiation patterns are considered. If this change were to be implemented, the possibility of a more accurate outcome is inevitable. Furthermore, These two mentioned improvements do complement each other since more sections in a floor allows for a more precise representation of WIFI travel patterns to be modeled.

We will now discuss one future iteration of this model with the same objective of maximizing the demand coverage but with a different approach of modeling. One of the questions we were initially drawn towards was whether or not WIFI could travel through the ceilings and floors of the library. Due to the thick concrete structure of the building, we assumed no WIFI could travel vertically, hence why we implemented a model that allocates routers for each floor individually. If we were to look further into the travel of WIFI signals between floors, we could potentially consider the vertical axis between the floors as a method of delivering WIFI. For the most accurate result, the model can be done in a 3-dimensional continuous space. This would add more complexity to the model but could also result in a more efficient method of allocating routers to meet demand coverage.

References

[1] Extreme, "outer data sheet - 3825i/e indoor access point.",[online; accessed 05-March-2019] `https://www.sfu.ca/content/sfu/itservices/technical/wireless/coverage_maps.html?fbclid=IwAR1sNDbQd1z6yT5cKPDZOpKu66I9altt4Z5E4QooI82mBZt6rl20TKsyBbA/`.

[2] SFU, "Coverage maps.", [Online;accessed05-March-2019]. `https://www.sfu.ca/content/sfu/itservices/technical/wireless/coverage_maps.html?fbclid=IwAR1sNDbQd1z6yT5cKPDZOpKu66I9altt4Z5E4QooI82mBZt6rl20TKsyBbA/`

[3] SFU, "SFU floor plans and locations directory.", [Online;accessed05-March-2019]. `https://www.lib.sfu.ca/facilities/rooms-spaces/floorplans//`

[4] Steam, "Steam hardware software survey: February 2019", [Online;accessed05-March-2019]. `https://store.steampowered.com/hwsurvey//`

[5] T. Lu, "Floor Plans: 3rd Floor - WAC Bennett Library", [Online; accessed 31-March-2019] `https://www.lib.sfu.ca/facilities/rooms-spaces/floorplans/bennett-floor-three`

[6] SFU IT Services, "WAC Bennett Library 2000 Level", [Online; accessed 31-March-2019] `https://www.sfu.ca/content/dam/sfu/itservices/technical/network_services/Wireless%20Coverage%20Maps/wacbennett2000.pdf`

[7] SFU IT Services, "WAC Bennett Library 3000 Level", [Online; accessed 31-March-2019] `https://www.sfu.ca/content/dam/sfu/itservices/technical/network_services/Wireless%20Coverage%20Maps/wacbennett3000.pdf`

[8] SFU IT Services, "WAC Bennett Library 4000 Level", [Online; accessed 31-March-2019] `https://www.sfu.ca/content/dam/sfu/itservices/technical/network_services/Wireless%20Coverage%20Maps/wacbennett4000.pdf`

[9] SFU IT Services, "WAC Bennett Library 5000 Level", [Online; accessed 31-March-2019] `https://www.sfu.ca/content/dam/sfu/itservices/technical/network_services/Wireless%20Coverage%20Maps/wacbennett5000.pdf`

[10] SFU IT Services, "WAC Bennett Library 6000 Level", [Online; accessed 31-March-2019] `https://www.sfu.ca/content/dam/sfu/itservices/technical/network_services/Wireless%20Coverage%20Maps/wacbennett6000.pdf`