# Intersection Optimization Analysis: Cambie \& W. Broadway, Vancouver 

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#### Abstract

The cross streets of W. Broadway and Cambie is one of the busiest intersections in the Vancouver Metropolitan area. Introducing a new traffic pattern, such as a diagonal crossing (scramble crossing), to improve pedestrian crossing efficiency can be risky considering the high traffic volume that the traffic corridor supports. The benefits from introducing a scramble crossing include the obvious shorter distance travelled by pedestrians when wanting to reach an opposite diagonal corner, and, given the right type of scramble crossing, a reduction of vehicle delays that are produced by walking pedestrians. Based on research and review of previous studies and methods, we constructed a mixed integer programming model to establish if the introduction of a scramble crossing at the chosen intersection would improve efficiency. Applying our model to the intersection of study generated positive results, allowing us to recommend that a scramble intersection be introduced based on our data.


## Introduction

The cross streets of Cambie and W. Broadway is one of the busiest intersections in the Vancouver district for both vehicles and pedestrians. This being the case makes it a complicated issue to improve the flow of one party without sacrificing the others efficiency.

One consideration for improving the overall flow is the introduction of a scramble intersection. A


Figure 1: Scramble intersection. (Mesa Architects photo, 2011) scramble intersection consists of an opportunity for pedestrians to have exclusive access to cross the intersection, while vehicles from all directions are at a halt.

Scramble intersections were implemented in Vancouver previously, in the 1950s, as one of the first cities in North America to have such a feature. It was later removed in 1970 due to capacity issues (Erin Loxam, 2011). They have now returned to the lower mainland in Richmond at No. 1 road and Moncton Street to promote pedestrian traffic in the area. Similarly, the City of Vancouver is also considering the idea at several intersections, one of which is the target of our study.

We conducted a survey on the streets, giving the public a chance to voice their opinions on the possibility of having a scramble intersection at Cambie Street and W. Broadway. We asked if they thought if a scramble intersection would make an improvement to the overall flow of both pedestrian and vehicle traffic. $65 \%$ people agreed that it would have a positive effect, while $26 \%$ disagreed and $9 \%$ remained uncertain (figure
2). A similar trend followed when asked if they think that it would be safe for pedestrians (figure 3). Overall, the majority of people supported the idea and thought it would have positive results. Some concerns people had were that it would confuse and delay elderly and disabled, and that introduction to a smaller intersection first may weed out any flaws in the system.

Figure 2


Figure 3


## Literature reviews

Rajnath Bissessar and Craig Tonde (Bissessar \& Tonder, 2008), working for the cities of Toronto and Calgary respectively, conducted studies on the methods used to implement scramble crossings and some effects it had on traffic. It covered the potential criterion that was used to classify whether an intersection be suitable for a scramble crossing:

- "High pedestrian volumes average $>$ 3,000 pedestrians per hour for an 8 hour period (Condition 1).
- Moderate pedestrian volumes average > 2000 pedestrians per hour for an 8 hour period (Condition 2).
- High turning vehicle volumes $>35 \%$ of total vehicular approach volume (Condition 3).
- High concentration of pedestrian-vehicle collisions $>3$ left turn and right-turn collisions where pedestrians had the right-of way over a three year period (Condition 4).
- There is a desire by at least $15 \%$ of pedestrians to cross diagonally (Condition 5).
- Unusual intersection geometry (5 or more legs) that preclude normal pedestrian crossing operation (Condition 6)."

The study also used the following validating combinations as a basis to justify the system implementation:

- "Condition 1.
- Conditions 2 and 3.
- Conditions 2 and 4.
- Conditions 2 and 5.
- Condition 6."

Once the crossing was deemed appropriate for the intersection, there was the issue of what design and pattern the crossing will take to ensure maximum effectiveness. There are 3 arrangements that are possible for a standard 4-leg intersection adopting a scramble intersection, each of which presented different advantages based on the intersection characteristics.

Upon making the decision to introduce the crossing, it was made sure that the crossings existence was to be obviously communicated to pedestrians and vehicles. This was one of the main issues, as bringing a new

concept like such, can be confusing to first time users. Many new signs were introduced and police officers were dispatched to direct all traffic for the initial stages.

Once the crossings had been implemented, results were compared to previous traffic patterns. As expected, delays for vehicle traffic and transit had significantly increased and brought the level of service, rated by motorists, down. However, the negative results were considered outweighed by the benefits to pedestrians and their larger numbers. Following this conclusion, more pedestrians have tended to use the intersection also whether it is for efficiency purposes or being a mild attraction for the


Figure 4: 3 possible types of vehicle and pedestrian movement when a scramble intersection imposed. (Bissessar \& Tonder, 2008)
city.

Having understood our general considerations and results expected from introducing a scramble intersection, we are now able to be more specific by understanding the crossing behavior of pedestrians and how it may be affected. Scholars such as Peter Gipps and B. Marksjo (1985), Aloys Borgers and Harry Timmermans (1986), and Kay Kitazawa and Michal Batty (2004) modeled pedestrian flow using route choices. They studied the comparison between variations in routes that a pedestrian could take from a starting point to a desired destination. It is similar to the idea of finding the shortest path, where pedestrians weight their preference between intermediary destinations and find the optimal path to the final destination. Queuing theory, developed by Erlang, is used broadly in analyzing pedestrian flow since the arrival rates of pedestrians at an intersection is a stochastic process, which is a key element. David Mitchell and James MacGregor Smith (2001) modeled pedestrian flow by analyzing pedestrians as a queuing network. They split intersections into components of a series and using queuing networks, were able to develop algorithms to analyse the performance of the network (Papadimitriou, Yannis, Golias, 2008).

Another article, by Di Sun, Dianhai Wang, Yongheng Chen, and Weiwei Guo (2011) also adapted queuing theory to pedestrian flows but built upon the signal pattern for pedestrians at intersections. By introducing formulae which considered parameters such as crowd densities, vehicle arrival and departure rates, a very accurate time for the length of pedestrian signals could be calculated.

One further study, done by Zengyi Yang (2010), consisted of developing an efficient procedure for constructing the signal plans in an isolated intersection that applied a genetic algorithm to optimize both vehicle and pedestrian flow through a studied intersection. The proposed procedure turned up to be as accurate as the Highway Capacity Software's algorithms, however more flexible. Upon analysis of his method, he concluded that diagonal crosswalks are most efficient when there is a large amount of traffic wanting to turn, and that the pedestrian crossing type should change from two-way to a scramble crossing when corresponding pedestrian and vehicle volumes grow considerably.

When there is not a large pedestrian use of an intersection, turning vehicles may also cause delays as Ray Schneider (2011) studied in western Ottawa using a video collection unit to determine the number of vehicles turning left in specific intersections. This data was used to optimize intersections in Ottawa by adjusting light patterns appropriately. Optimizing signal lights in heavy traffic intersections produces many benefits, mainly those that reduce the fuel consumption and emissions of carbon dioxides which have negative implications on the environment. Improving light cycles will allow clearance time for vehicles to be faster by minimizing their waiting time in the queue with relatively small costs to the city, compared to extensive construction projects.

After imposing improved traffic cycles, the city of Ottawa achieved significant results. With a 12.5:1 cost-benefit ratio, they gained an annual saving of $\$ 5$ million. More importantly, since vehicles reduced their idle time, there has been savings of over 273,000 gallons of fuel annually, reducing harmful air pollutants by 1000 tons.

The criteria provided by Rajnath Bissessar and Craig Tonde will give us a preliminary expectation as to whether a scramble intersection, and in turn a shorter cycle length, will be a positive implementation at our intersection of study, while methods adopted by other papers will help to model our problem and obtain a result based on data.

## Objective

We aim to find out if a properly introduced scramble intersection, along with adjusted traffic light times, will improve the overall flow primarily for pedestrians and secondarily for vehicles through the intersection of Cambie and Broadway during rush hour traffic.

Introducing a scramble intersection will allow for pedestrians to take the shortest distance when travelling across the intersection diagonally, giving them the possibility to save time overall. Vehicles will also have a chance to save time given that they will not be delayed when turning due to crosswalk use. However, these benefits do not happen concurrently and appropriate traffic light times and signal patterns must be established to optimize the results.

## Problem

The current vehicle traffic pattern, at the Cambie and Boadway intersection, consists of multiple lanes in four directions (North-South, and East-West) with dedicated left turn lanes for both directions along W. Broadway. Traffic lights allow for left-turn advance both ways (along W.

Broadway), left turn advance and straight through in one direction (also along W. Broadway), or straight through each way. Outside lanes along W. Broadway are for parking (until close to the intersection where they are used for right turns) except during rush hour periods where it is an HOV lane, usable only by busses. Current vehicle delays emanate from turning lanes being restricted by pedestrians crossing.


Figure 5: Current vehicle and pedestrian flows.

The pedestrian pattern is a standard crosswalk moving parallel with vehicle traffic when it is travelling straight through the intersection. However, the majority of the pedestrians are arriving at the north-west corner, where many businesses reside, and the south-east corner, where a major Skytrain station is located.

| Corner | Average \# of <br> pedestrians during 1 <br> minute intervals | Pedestrian arrivals / <br> second |
| :---: | :---: | :---: |
| North-West | 19.2 | 0.32 |
| North-East | 10.2 | 0.17 |
| South-West | 9.6 | 0.16 |
| South-East | 19.2 | 0.32 |

From the above data, we can calculate the average number of pedestrians
to arrive per hour to be 3510 . Using the City of Torontos criteria (Bissessar \& Tonder, 2008), stated earlier, we will justify the consideration of a scramble intersection by condition 1: "High pedestrian volumes average > 3,000 pedestrians per hour for an eight hour period."

To summarize, there are 4 components of the system: vehicle traffic pattern, pedestrian pattern, vehicle traffic control policy, and pedestrian crossing control policy. Currently, we believe that the components may not be set accordingly to achievethe optimal overall flow for pedestrians and vehicles travelling through the intersection.

Since the intersection has varying levels of activity throughout the day, our primary concern will be the evening rush hour period (3:00pm to 6:00pm) which adopts the HOV lanes. We will be adopting the transportation hierarchy used by the Vancouver City Council ("Traffic management", 2010), which is as follows: pedestrian first, then bicycles, transit, movement of goods and vehicular traffic. However, considering the very low volume of bicycle and movement of goods traffic that was observed at this particular intersection, those will be merged with pedestrians and vehicles respectively, given that they use equal routes.

## Solution

We aim to develop a linear programming model that can be applied to the target intersection (and can be modified for different intersections by adjusting the appropriate parameters) to determine if using a scramble crosswalk is beneficial over current methods. For the purposes of our problem, the scramble intersection of Type 1 (from figure 4) will be used
to model the intersection. This has been chosen because it represents a potential solution for current problems that arise from vehicles wanting to turn right while pedestrians travel across the intersection.


Figure 6: Type 1. (Bissessar \& Tonder, 2008)

Our approach will be to focus primarily on the pedestrian wait times between permissible signals, as well as the wait times for vehicles. Constraints to follow will include making sure all traffic light patterns are of sufficient length to allow for all cars to travel through the intersection, as well as making sure the overall cycle is not too long. With the solution we will be able to recommend the optimal cycle of traffic lights and crossing patterns to optimize the overall flow.

Once the optimal cycle has been determined, we will compare the total cycle length to the existing cycle length. If the scramble crossing adapted cycle length is shorter than the existing length, we will deem the implementation as an appropriate decision. By having a shorter cycle length, benefits will be shown through less fuel consumption and impact on the environment as well as improving overall travel time for vehicles and pedestrians.

## Modelling

Our initial model will be constructed to find a feasible traffic light cycle given that a scramble crossing will be implemented. Since pedestrians are at the top of the traffic hierarchy as adopted from Vancouver city council ("Traffic management", 2010), our objective function will be to minimize the end time of the final traffic light pattern in a cycle. In other words, minimize the maximum time pedestrians will have to wait for a chance to cross. Following that solution, we will then address the next level in the hierarchy, being vehicles, and arrange the feasible light cycle in a way such that maximum vehicle wait times for each direction are minimized.

The following assumptions will be made when finding our feasible light cycle:

1. Vehicle and pedestrian arrival rates are constant, as an average over the studied time period.
2. Service rates are constant.
3. We will be looking at this intersection alone and not considering surrounding ones ortheir effects.
4. There is no initial queue at the intersection.
5. Cars wanting to turn right cannot turn when blocked by perpendicular traffic travelling straight.
6. When analyzing the accumulation of vehicles, those going straight are blocking both left and right turn lanes so all arrival rates emanating from that direction contribute to the line-up.
7. Traffic light pattern lengths will include the amber light segment as vehicles generally continue to proceed through it at an increased rate as
if the traffic light were green.
8. The time it takes for vehicles to be travelling at a constant rate is linear.
9. It takes no time for turning vehicles to reach a constant service rate as service rates are very small for such traffic patterns.
10. All corners have an equal maximum capacity for pedestrians.

Constraints that must be met for a feasible traffic light cycle:

1. Each traffic light pattern will happen at most once.
2. The scramble crossing traffic light pattern will happen exactly once.
3. Each possible traffic light pattern must happen one at a time (if at all) to prevent collisions from happening.
4. All cars that arrive during the cycle time must all make it through the intersection, to prevent line ups from carrying over to the next cycle and building upon each other.
5. Line-ups of traffic in each direction cannot to exceed a certain length. For example, the line-up cannot reach back to another intersection.
6. Each corner for pedestrians has a maximum capacity that cannot be exceeded, otherwise overflow into the streets may occur.
7. If a traffic light pattern exists, it will meet a minimum length.

Parameters for the problem, which may vary depending on the application, include:

1. Predetermined time for length of scramble crossing traffic light since time it takes pedestrians to walk across intersection is constant.
2. An upper bound for the total cycle length.
3. Average length of a car.
4. Minimum job length desired.
5. Time for vehicles to reach a constant service rate.
6. Service rates of traffic lights in each direction going straight, turning left or turning right.
7. Arrival rates of cars in each direction going straight, turning left or turning right.
8. Maximum permitted length for line-up to build to (assuming majority of cars are going straight so
9. Pedestrian arrival rates at each corner.
10. Maximum pedestrian capacity at each corner.

For the design of our model, we have split up traffic light patterns into various Jobs which consist of each permutation of traffic light pattern possible at a 4-leg intersection (see figure 7).


Figure 7: All possible "Jobs" in a 4-leg intersection.

## Decision variables

We introduce three sets of decision variables: $\left\{l_{i}\right\},\left\{x_{i j}\right\}$, and $\left\{y_{i k}\right\}$. $l_{i}$ is an non-negative integer variable, it stands for length of Job $i, i \in$ $\{1,2, \ldots, 8\} ; x_{i j}$ is a binary variable defined for $i \in\{1,2, \ldots, 9\}, j \in\{1,2, \ldots, U\}$ in the following way:

$$
x_{i j}= \begin{cases}1, & \text { if Job } i \text { starts during time } j \\ 0, & \text { otherwise }\end{cases}
$$

$y_{i k}$ is a binary variable defined for $i \in\{1,2, \ldots, 9\}, k \in\{1,2, \ldots, 9\}$, and $i<k$ in the following way:

$$
y_{i k}= \begin{cases}1, & \text { if Job } i \text { ends before Job } k \\ 0, & \text { otherwise }\end{cases}
$$

For our convinience, we introduce another set of dependant variables: $\left\{s_{i}\right\}$ and $\left\{e_{i}\right\}$, where $i \in\{1,2, \ldots, 9\}: s_{i}$ stands for starting time of Job $i$, $e_{i}$ stands for ending time of Job $i$. Simple linear equations define $s_{i}$ and $e_{i}$ :

$$
\begin{aligned}
s_{i} & =\sum_{j=1}^{U} j \times x_{i j} \\
e_{i} & =s_{i}+l_{i}
\end{aligned}
$$

Finally, we define $E$ as the end of the last Job.

## Model Parameters

The following parameters are introduced to make the model more flexible:

- $l_{9}$ : length of job 9
- $U$ : upper bound for cycle time
- LE: average length of car
- $W$ : A large constant (to be larger than all Job lengths)
- $M$ : Minimum Job length
- $S$ : The time it takes for vehicles to reach a constant service rate

Group A. Service rates of vehicles going in different directions:

- $S_{S}$ : service rate of vehicles going straight per lane
- $S_{L}$ : service rate of vehicles going left per lane
- $S_{R}$ : service rate of vehicles going right per lane

Group B. Arrival rates of vehicles travelling in different directions:

- $A_{W S}$ : arrival rate of vehicles going straight from westbound (on Broadway)
- $A_{W L}$ : arrival rate of vehicles turning left from westbound (on Broadway)
- $A_{W R}$ : arrival rate of vehicles turning right from westbound (on Broadway)
- $A_{E S}$ : arrival rate of vehicles going straight from eastbound (on Broadway)
- $A_{E L}$ : arrival rate of vehicles turning left from eastbound (on Broadway)
- $A_{E R}$ : arrival rate of vehicles turning right from eastbound (on Broadway)
- $A_{N S}$ : arrival rate of vehicles going straight from northbound (on Cambie)
- $A_{N L}$ : arrival rate of vehicles turning left from northbound (on Cambie)
- $A_{N R}$ : arrival rate of vehicles turning right from northbound (on Cambie)
- $A_{S S}$ : arrival rate of vehicles going straight from southbound (on Cambie)
- $A_{S L}$ : arrival rate of vehicles turning left from southbound (on Cambie)
- $A_{S R}$ : arrival rate of vehicles turning right from southbound (on Cambie)

Group C. Maximum permitted line-up for each direction:

- $L_{W}$ : length of permitted line up allowed for westbound (on Broadway)
- $L_{E}$ : length of permitted line up allowed for eastbound (on Broadway)
- $L_{N}$ : length of permitted line up allowed for northbound (on Cambie)
- $L_{S}$ : length of permitted line up allowed for southbound (on Cambie)

Group D. Arrival rates of pedestrians going at each corner:

- $P_{N W}$ : arrival rate of pedestrians at North-West corner
- $P_{N E}$ : arrival rate of pedestrians at North-East corner
- $P_{S W}$ : arrival rate of pedestrians at South-West corner
- $P_{S E}$ : arrival rate of pedestrians at South-East corner
- $P_{M A X}$ : Maximum permitted number of pedestrians standing at each corner


## Objective function

## Minimize $E$

That is, minimize the end time of the last Job, which will be the cycle length and the time pedestrians will have to wait between each crossing.

## Constraints

1. Jobs 1 through 8 must be completed at most once:

$$
\sum_{j=1}^{U} x_{i j} \leq 1, \text { for } i \in\{1,2, \ldots, 8\}
$$

2. Job 9 (scramble crosswalk) must be completed once:

$$
\sum_{j=1}^{U} x_{9 j}=1
$$

3. One Job happening at a time:
$e_{i} \leq s_{k}+y_{i k} * W$, and $e_{k} \leq s_{i}+\left(1-y_{i k}\right) * W$, for $i, k \in\{1,2, \ldots, 9\}, i<k$
4. Traffic service rate must be higher than arrival rate, i.e. total cars serviced in each direction must be greater than the total number of cars accumulating in that direction during the entire cycle (note that for straight directions the number of cars that will not be serviced due to gradual acceleration through intersection must be subtracted; Broadway serves 2
straight lanes in each direction, Cambie serves 3 straight lines in each direction):
a. Straight on Eastbound Broadway: $\left(l_{1} * 2+l_{2} * 2-2 * S * \frac{1}{2}\right) * S_{S} \geq$ $A_{E S} * \sum_{i=1}^{9} l_{i}$
b. Straight on Westbound Broadway: $\left(l_{1} * 2+l_{3} * 2-2 * S * \frac{1}{2}\right) * S_{S} \geq$ $A_{W S} * \sum_{i=1}^{9} l_{i}$
c. Straight on North Cambie: $\left(l_{5} * 3-3 * S * \frac{1}{2}\right) * S_{S} \geq A_{N S} * \sum_{i=1}^{9} l_{i}$
d. Straight on South Cambie: $\left(l_{5} * 3-3 * S * \frac{1}{2}\right) * S_{S} \geq A_{S S} * \sum_{i=1}^{9} l_{i}$
e. Left from Eastbound Broadway: $\left(l_{2}+l_{4}\right) * S_{L} \geq A_{E L} * \sum_{i=1}^{9} l_{i}$
f. Left from Westbound Broadway: $\left(l_{3}+l_{4}\right) * S_{L} \geq A_{W L} * \sum_{i=1}^{9} l_{i}$
g. Right from Eastbound Broadway: $\left(l_{1}+l_{2}+l_{4}\right) * S_{R} \geq A_{W R} *$ $\sum_{i=1}^{9} l_{i}$
h. Right from Westbound Broadway: $\left(l_{1}+l_{3}+l_{4}\right) * S_{R} \geq A_{E R} *$ $\sum_{i=1}^{9} l_{i}$
i. Right from North Cambie: $\left(l_{3}+l_{4}+l_{5}\right) * S_{R} \geq A_{S R} * \sum_{i=1}^{9} l_{i}$
j. Right from South Cambie: $\left(l_{3}+l_{4}+l_{5}\right) * S_{R} \geq A_{N R} * \sum_{i=1}^{9} l_{i}$
5. Cars queuing cannot exceed length allowed for lineup, i.e. all Jobs
where vehicles are not travelling through the intersection contribute to the accumulation of a line-up that cannot exceed a predetermined length:

$$
\begin{aligned}
& \left(l_{3}+l_{4}+l_{5}+l_{9}\right) *\left(A_{E S}+A_{E L}+A_{E R}\right) * L E \leq L_{E} \\
& \left(l_{2}+l_{4}+l_{5}+l_{9}\right) *\left(A_{W S}+A_{W L}+A_{W R}\right) * L E \leq L_{W} \\
& \left(l_{1}+l_{2}+l_{3}+l_{4}+l_{9}\right) *\left(A_{N S}+A_{N L}+A_{N R}\right) * L E \leq L_{N} \\
& \left(l_{1}+l_{2}+l_{3}+l_{4}+l_{9}\right) *\left(A_{S S}+A_{S L}+A_{S R}\right) * L E \leq L_{S}
\end{aligned}
$$

6. Number of pedestrians arriving at each corner during the entire traffic light cycle (minus pedestrian crossing Job) must be less than maximum permitted number of pedestrians at each corner:

$$
\begin{aligned}
& P_{N W} \sum_{i=1}^{8} l_{i} \leq P_{M A X} \\
& P_{N E} \sum_{i=1}^{8} l_{i} \leq P_{M A X} \\
& P_{S W} \sum_{i=1}^{8} l_{i} \leq P_{M A X} \\
& P_{S E} \sum_{i=1}^{8} l_{i} \leq P_{M A X}
\end{aligned}
$$

7. If a Job exists, its length will be at least 8 seconds and if it does not exist, it will have length of 0 :

$$
l_{i} \leq \sum_{j=1}^{U} x_{i j} * W, \text { and } l_{i} \geq \sum_{j=1}^{U} x_{i j} * M, \text { for } i=\{1,2, \ldots, 9\}
$$

8. All Jobs ending times are less than or equal to the maximum of
the Jobs ending times, $E$.

$$
E \geq e_{i}, \text { for } i=\{1,2, \ldots, 9\}
$$

## Study Specific Parameters

- Jobs 6, 7, and 8 are not considered since they compromise the current system.
- The length of Job 9 is calculated using the diagonal distance across the intersection, the average walking speed of an individual, and the pedestrian walk interval duration. Using the distance across current crosswalks (approximately 72 ft and 94 ft ), the diagonal distance was found to be approximately 118 ft . Adopting the Signal Timing Manuals (Koonce, 2008), average walking speed of 4 feet/second, we can conclude that the time to walk the diagonal distance is 29 seconds. This will account for part of the Job, which is the "nowalk time" where pedestrians have a chance to finish walking if they have started. In addition, the pedestrians must have a chance to start walking, once they have been given the signal to proceed. For this time, we follow the Signal Timing Manuals, "Pedestrian walk interval duration" table:

Our intersection of study falls under a "high pedestrian volume area", so for this portion of the Job length, we chose 10, to keep the Job length at a minimum.

$$
l_{9}=39
$$

- The upper bound of our cycle time was set to be 180 seconds. This is higher than the current cycle time of 120 seconds, and allows us

| Conditions | Walk Interval Duration (PW), s |
| :--- | :---: |
| High pedestrian volume areas (e.g., school, central <br> business district, sports venues, etc.) | 10 to 15 |
| Typical pedestrian volume and longer cycle length | 7 to 10 |
| Typical pedestrian volume and shorter cycle length | 7 |
| Negligible pedestrian volume | 4 |
| Conditions where older pedestrians are present | Distance to center of road divided by 3.0 feet per second |

Figure 8: Pedestrian walk interval durations (Koonce, 2008)
to look for restricting factors if our solution does not show to be better than current system.

$$
U=180
$$

- The average length of a car was taken as the length of a 2012 Honda Civic. This car provides a fair representation of the average car size in Vancouver traffic.

$$
L E=4.5 \mathrm{~m}
$$

- W is a large constant required to be larger than all Job lengths:

$$
W=10000
$$

- Our minimum Job length was set to 8 , giving sufficient time for vehicles to build up speed such that there is consistent service rates:

$$
M=8
$$

- Time it takes for vehicles to reach constant service rate was set from observation:

$$
S=8
$$

- The following service rates per lane were gathered from a typical intersection as per the amount of cars consecutively travelling through in a set time frame:

$$
\begin{aligned}
S_{S} & =1.25 \\
S_{L} & =0.7 \\
S_{R} & =0.4
\end{aligned}
$$

- The following arrival rates were calculated by first calculating the ratio of cars wishing to travel in each direction along each route, and applying this to the Vanmap statistics of hourly traffic rates (2011) for per second rates of arrival:

| Parameter <br> (direc- <br> tion) | Cars <br> counted / <br> Total cars <br> going that <br> direction <br> hours period <br> of study <br> (from <br> Vanmap) | Total number <br> cars/second | Arrival <br> rate: |
| :---: | :---: | :---: | :---: |
| $A_{W S}$ | $154 / 232$ | 3457 | 0.21 |
| $A_{W L}$ | $52 / 232$ | 3457 | 0.07 |
| $A_{W R}$ | $26 / 232$ | 3457 | 0.04 |
| $A_{E S}$ | $223 / 277$ | 3539 | 0.26 |
| $A_{E L}$ | $40 / 277$ | 3539 | 0.05 |
| $A_{E R}$ | $14 / 277$ | 3539 | 0.02 |
| $A_{N S}$ | $247 / 271$ | 3531 | 0.30 |
| $A_{N R}$ | $24 / 271$ | 3531 | 0.03 |
| $A_{S S}$ | $212 / 240$ | 5038 | 0.41 |
| $A_{S R}$ | $28 / 240$ | 5038 | 0.05 |

- The permissible length of car accumulation is the distance to the next intersection (note that $L_{S}$ is large because of the bridge in this direction):

$$
\begin{gathered}
L_{W}=110 \\
L_{E}=170 \\
L_{N}=100 \\
L_{S}=1000
\end{gathered}
$$

- Arrival rates for pedestrians were calculated as an average of how many pedestrians arrive over a 10 minute period at each corner:

| Parameter | Total over 10 minutes | Pedestrians/second |
| :---: | :---: | :---: |
| $P_{N W}$ | 191 | 0.32 |
| $P_{N E}$ | 100 | 0.17 |
| $P_{S W}$ | 93 | 0.16 |
| $P_{S E}$ | 189 | 0.32 |

- Maximum capacity for pedestrians at each corner is set to 20:

$$
P_{M A X}=20
$$

## Study Results

Using Excel Open Solver, the solution for the linear programming model was obtained. The results indicate that introducing a scramble intersection will yield an optimal cycle length of $\mathbf{7 1}$ seconds. The optimal Job schedule is summarized in the following table:

| Job: | Order: | Start time: | Length: | End time: |
| :---: | :---: | :---: | :---: | :---: |
| Job 1 | 2 | 39 | 12 | 51 |
| Job 2 | - | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Job 3 | - | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Job 4 | 4 | 63 | 8 | 71 |
| Job 5 | 3 | 51 | 12 | 63 |
| Job 9 | 1 | 0 | 39 | 39 |

A comparison of the resulting optimal cycle length against the existing cycle length of 79 seconds indicates that a scramble crossing (based of the criteria assumed) can be recommended as the project currently stands.

To further strengthen our recommendation, given additional time some further research of parameters and constraints would allow for more accurate results. One area would include further analysis of traffic lights service rates, as they are affected by intersection length and the gradual increase in the speed that traffic moves. Di Sun, Dianhai Wang, Yongheng Chen, and Weiwei Guo (2011) introduced accurate formulae for calculating the optimal signal lengths for pedestrian walk time which, given time and to collect appropriate data, would allow for stronger evidence on proposed length of the scramble crossing pedestrian signal.

Some intersection signals are co-ordinated with other nearby intersections and implementing this new cycle may disrupt the flow of traffic through such areas. Further studies may include introduction of a scramble crossing at certain intersections, while optimizing the system as a whole and considering the relationships between connected traffic light cycles. Similarly, allowing the signal times to adapt to different times of the day
where traffic patterns change would allow for more efficient use of the intersection.

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