Novelty & Nutrition

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Abstract

This paper proposes and solves a modified diet problem that looks at the consumption of food from a larger perspective. Here we try to maximize the nutrient intake of an individual, as well as their preference for not eating the same food for every meal and minimizing the cost of purchasing their food. This problem was formulated as a linear program and solved using Open Solver in Microsoft Excel.

Introduction

This model consists of optimizing 3 conditions:

1. Cost should be minimized
2. Nutrition requirements should be met
3. Novelty should be maximized

Cost is measured by adding up the cost to buy the amount of food eaten at each meal. This value is then multiplied by a scaling constant to ensure that comparisons are fair between the 3 objectives (cost is not expected to exceed \$150 whereas novelty will have a maximum of 210; more on this later).

Nutrition is measured by determining if the required daily intake for each nutrient is met, and assigning a bonus to the objective if it is met. The value of this bonus is determined so as to keep the maximum value of each objective approximately constant. Furthermore, a minimum number of nutrient requirements that must be met on each day is imposed, as well as a minimum number of days on which each nutrient requirement must be met. This is to ensure that our hypothetical person does not die from an acute lack of a nutrient that is particularly expensive to obtain compared to the others. Not all nutrient requirements are required to be met on every day because it is not uncommon for a person to go for a day or two without getting enough of a particular nutrient.

Novelty is measured for a particular meal and a particular food by assigning a novelty score to each of the previous meals (starting at 6 and decreasing uniformly to 1 with each step backward). A sum is then computed for the meal-food combination being considered by adding together all novelty scores of previous meals with which that particular food was eaten. This sum is then computed for every possible meal-food combination and these are all added together to obtain the model's novelty score.

Certain restrictions are imposed on the amount that can be eaten for each meal. The amount of food of each type eaten with each meal is restricted to be below a certain value to prevent meals consisting of only 1 type of food. Furthermore, the amount of food consumed with each breakfast, lunch and dinner must fall within a certain range (with minimum and maximum values increasing respectively (ie. breakfast $<$ lunch $<$ dinner). Finally, if a particular food is eaten with a meal, then a minimum amount of that food must be eaten (this prevents meals that consist of 0.5 grams of steak).

The significance of this model is that it addresses a gap in the structure of most diet models. The original diet problem model consisted of meeting nutrient requirements while minimizing costs ("History of the Diet Problem", reference 1) and many other diet problem models discussed in class either did not consider novelty or imposed it through the use of constraints. This model does not require that the individual eat different foods with every meal but instead the novelty score models a person's 'preference' to eat different foods.

Preliminaries

Some information is required before this model can be discussed:

- Nutrient requirements are for a 19-30 year old female (requirements obtained from the document "Dietary Reference Intakes: Estimated Average Requirements", reference 2).
- 1 unit of food is equivalent to 100 grams of edible portion (as defined in the Canadian Nutrient File, reference 3).
- Prices for foods are from local grocery stores (Save-on-Foods and Safeway).
- The objective in this model measures negativity (ie. lack of novelty, money spent), so the bonus obtained from meeting nutrition requirements is made negative when included in the objective. This implies that the objective function is minimized for this model.
- Indices are as follows:
    - i indexes meals ($i \in [1, 42]$)
    - j indexes foods ($j \in [1, 20]$)
    - k indexes nutrients ($k \in [1, 15]$)
    - m indexes days ($m \in [1, 14]$)

Variables

Let $B_{i,j}$ be the number of units of food j eaten with meal i. These are the only true decision variables in is model.
Let L_i be the amount of food eaten with meal i.
Let $F_{j,m}$ be the amount of food j eaten on day m.
Let $E_{k,m}$ be the amount of nutrient k obtained on day m.
Let $H_{k,m}$ equal 0 if requirements for nutrient k are met on day m and 1 otherwise.
Let $X_{k,m}$ be the percent that day m is short of the requirement for nutrient k.
Let $K_{i,j}$ equal 1 if food j is eaten with meal i and 0 otherwise. Defined to be 0 for $i \leq 0$ for convenience of writing constraints.
Let $N_{i,j}$ be the lack of novelty penalty associated with food j at meal i.
Let $Z_{i,j}$ be the lack of novelty penalty actually obtained through food j with meal i (zero if food j is not eaten).

Data

Let $A_{j,k}$ be the amount of nutrient k in one unit of food j.
Let R_k be the required daily intake for nutrient k.
Let C be the bonus for meeting the requirement of 1 nutrient on 1 day (this value is positive for ease of interpretation).
Let D_j be the cost of 1 unit of food j.
Let S_1 be the scaling constant for cost.
Let S_2 be the scaling constant for novelty.

Objective Function

The sum of the effects of lack of nutrition, money spent and lack of novelty is to be minimized. Equivalently, minimize

$$\overbrace{\left[\sum_{k=1}^{15}\sum_{m=1}^{14}(-C \cdot (1 - H_{k,m}))\right]}^{\text{Bonus from meeting nutrient requirements (note } C \text{ is now made negative).}} +$$

$$\overbrace{\left[S_1 \cdot \sum_{i=1}^{42}\sum_{j=1}^{20}(D_j \cdot B_{i,j})\right]}^{\text{Total cost of food consumed}} +$$

$$\overbrace{\left[S_2 \cdot \sum_{i=1}^{42} \sum_{j=1}^{20} Z_{i,j} \right]}^{\text{Model's lack of novelty score}}$$

Constraints

By definition, the amount of food eaten with a particular meal is the sum of the amount of food of each type eaten with that meal. Equivalently,

$$\forall j : L_j = \sum_{i=1}^{20} B_{i,j}$$

The amount of food of each type eaten with each meal must not exceed a specified value. Equivalently,

$$\forall i \forall j : B_{i,j} \leq 5$$

The amount of food eaten for each breakfast, lunch and dinner must be within a specified range. Equivalently,

$$\forall m : 2 \leq L_{3m-2} \leq 5$$
$$\forall m : 4 \leq L_{3m-1} \leq 7$$
$$\forall m : 6 \leq L_{3m} \leq 10$$

Note that the formulation of this constraint requires that $i = 1$ (meal 1) corresponds to a breakfast.

The amount of food j that is eaten on day m is the sum over all meals eaten that day of how much of food j is eaten at that meal. Equivalently,

$$\forall j \forall m : F_{j,m} = \sum_{i=3m-2}^{3m} B_{i,j}$$

The amount of nutrient k obtained on day m is the sum over all foods of how much of that food was eaten times its nutrition content for nutrient k. Equivalently,

$$\forall k \forall m : E_{k,m} = \sum_{j=1}^{20} (F_{j,m} \cdot A_{j,k})$$

The percent that nutrient k is short of its required level on day m is the required amount minus the obtained amount, all divided by the required amount. Equivalently,

$$\forall k \forall m : X_{k,m} = \frac{R_k - E_{k,m}}{R_k}$$

Each nutrient requirement is exceeded by at most some specified value. Equivalently,

$$\forall k \forall m : X_{k,m} \geq -6.5$$

Each $H_{k,m}$ is a binary variable that is 0 if the required intake level for nutrient k is met on day m and 1 otherwise. Equivalently,

$$\forall k \forall m : H_{k,m} \in \{0, 1\}$$
$$\forall k \forall m : X_{k,m} \leq H_{k,m}$$

At least 12 nutrients must be met on each day. Equivalently,

$$\forall m : \sum_{k=1}^{15} H_{k,m} \leq 3$$

Note that this sum must be less than or equal to 15-12 because H is 0 when the nutrient requirement is met.

Each nutrient requirement must be met on at least 11 of the 14 days. Equivalently,

$$\forall k : \sum_{m=1}^{14} H_{k,m} \leq 3$$

Each $K_{i,j}$ is a binary variable that is equal to 1 if food j is eaten with meal i and 0 otherwise. Equivalently,

$$\forall i \forall j : K_{i,j} \in \{0, 1\}$$
$$\forall i \forall j : B_{i,j} \leq M \cdot K_{i,j}$$

Where M is some very large number. Note that $K_{i,j} = 0$ for $i \leq 0$.

If food j is eaten with meal i then at least 1 unit of it must be eaten. Equivalently,

$$\forall i \forall j : B_{i,j} \geq K_{i,j}$$

The utility lost from eating a food that was eaten n meals ago is $7-n$ where $1 \leq n \leq 6$. Equivalently,

$$\forall i \forall j : N_{i,j} = \sum_{n=i-6}^{i-1} [(i-n) \cdot K_{j,n}]$$

Each $Z_{i,j}$ behaves as though it is the product of the corresponding $N_{i,j}$ and $K_{i,j}$. Equivalently,

$$\forall i \forall j : Z_{i,j} \geq N_{i,j} + 21K_{i,j} - 21$$
$$\forall i \forall j : Z_{i,j} \leq N_{i,j}$$
$$\forall i \forall j : Z_{i,j} \leq 21k_{i,j}$$

Each $B_{i,j}$ and $Z_{i,j}$ is non-negative. Equivalently,

$$\forall i \forall j : B_{i,j} \geq 0$$
$$\forall i \forall j : Z_{i,j} \geq 0$$

Model Formulation

Maximize:

$$\sum_{k=1}^{15} \sum_{m=1}^{14} (-C \cdot (1 - H_{k,m})) + S_1 \cdot \sum_{i=1}^{42} \sum_{j=1}^{20} (D_j \cdot B_{i,j}) + S_2 \cdot \sum_{i=1}^{42} \sum_{j=1}^{20} Z_{i,j}$$

Subject to the following constraints:

$$\forall j : L_j = \sum_{i=1}^{20} B_{i,j}$$
$$\forall i \forall j : B_{i,j} \leq 5$$
$$\forall m : 2 \leq L_{3m-2} \leq 5$$
$$\forall m : 4 \leq L_{3m-1} \leq 7$$
$$\forall m : 6 \leq L_{3m} \leq 10$$
$$\forall j \forall m : F_{j,m} = \sum_{i=3m-2}^{3m} B_{i,j}$$
$$\forall k \forall m : E_{k,m} = \sum_{j=1}^{20} (F_{j,m} \cdot A_{j,k})$$
$$\forall k \forall m : X_{k,m} = \frac{R_k - E_{k,m}}{R_k}$$
$$\forall k \forall m : X_{k,m} \geq -6.5$$
$$\forall k \forall m : H_{k,m} \in \{0, 1\}$$
$$\forall k \forall m : X_{k,m} \leq H_{k,m}$$
$$\forall m : \sum_{k=1}^{15} H_{k,m} \leq 3$$
$$\forall k : \sum_{m=1}^{14} H_{k,m} \leq 3$$

$$\forall i \forall j : K_{i,j} \in \{0, 1\}$$
$$\forall i \forall j : B_{i,j} \leq M \cdot K_{i,j}$$
$$\forall i \forall j : B_{i,j} \geq K_{i,j}$$
$$\forall i \forall j : N_{i,j} = \sum_{n=i-6}^{i-1} [(i - n) \cdot K_{j,n}]$$
$$\forall i \forall j : Z_{i,j} \geq N_{i,j} + 21K_{i,j} - 21$$
$$\forall i \forall j : Z_{i,j} \leq N_{i,j}$$
$$\forall i \forall j : Z_{i,j} \leq 21k_{i,j}$$
$$\forall i \forall j : B_{i,j} \geq 0$$
$$\forall i \forall j : Z_{i,j} \geq 0$$

Solution

After reviewing the data, the scaling values were set as follows.

Variable	Value
S_1	1
S_2	0.4
C	1.2

This model was solved using the Open Solver add-on in Microsoft Excel (reference 4). The solving process was stopped prematurely after 15 hours of running on the math department's supercomputer and the resulting solution is assumed to be optimal. The obtained objective function value is -23.71063948.

The solution did not meet nutrient requirements for Vitamin-A or Choline on 3 of the 14 days (no more than 1 nutrient was lacking on any particular day) and all other nutrient requirements were met on every day.

The menu that Open Solver chose is too large to include here. The menu for a sample day is be included for illustration purposes.

Breakfast	Red Pepper
	Chicken Breast
	Multigrain Bread
	Red Apple
	Canned Tuna
	Chocolate Granola Bar
Lunch	Tomato
	White Bread
Dinner	Tomato
	Salmon
	Fruit Yogourt
	Tortilla Chips

It is interesting to note that each food is eaten at least once in the full solution. Furthermore, some chosen lunches are larger than the dinners. This occurs because of the overlapping meal size restrictions on lunches and dinners and was deemed acceptable by the authors.

References

Reference 1:	Obtained from http://www.statslab.cam.ac.uk/~rrw1/opt/diet_history.html
Reference 2:	Obtained from the National Agriculture Library, United States Department of Agriculture
Reference 3:	Obtained from http://www.hc-sc.gc.ca/fn-an/nutrition/fiche-nutri-data/index-eng.php
Reference 4:	Open Solver obtained from http://www.opensolver.org/