Diet Problem: “Healthy” Fast food

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Abstract

In this paper we consider the specific diet problem, which comes from the real life situation: an average student chooses a two weeks diverse lunch plan based on food available on campus. A mixed integer programming model is used to find an optimal lunch plan, so that Daily Recommended Intake is satisfied and consumption of harmful nutrients is minimized. The results obtained for a specific case of SFU Surrey Campus were found to be very reasonable.

Introduction

Diet problem is considered to be one of the huge life-long topics for every person to be concerned about all over the world. People are trying to eat as healthily as possible as a part of their health lifestyle. To be more related to the audience, we have narrowed down the target group of the project to be moderately active college students.

The data sets of this project about various types of nutrients and the boundaries of the constraints are from the restaurants official websites and some well-known information websites about recommended daily nutrition needs and limits. By recognizing a variety of assumptions for the model including the setting of targets breakfast and dinner, we have used some special ratios to covert these bounds into the boundaries for lunches. The constraints are set up based on all the information and the objective function is established using different penalties for each element based on the average intake correspondingly. Excel OpenSolver is utilized to solve this linear programming problem after all the data are inputted into Excel. After many hours of running of the Excel OpenSolver, an optimal fourteen-day lunch plan is reached with the nutrition intakes on a daily basis based on the model as well as a minimum total intake of excess calorie, sodium, cholesterol, trans fat, and total fat.

The method used in this project shows an approach to deal with diet meal plans for people with busy and fast-paced lifestyle. It can be expanded to focus on different types of target groups, to add breakfasts and dinners into consideration, and to change the number or name of fast-food restaurants by altering the input data sets of the corresponding information. We believe that this presents a great down-to-earth solution to the diet problem.

This paper will provide detailed explanations about this specific diet problem, the assumptions made for the model, the model itself, and the outputs and results for our problem.

Background

Throughout the history of linear programming, the diet problem has played a really important role. First to be mentioned is the Stigler Diet, an optimization problem named after a US Economist George Stigler. He developed a diet problem to measure how much a normal person should choose from 77 types of food to eat so as to have a minimum cost based on the situation in 1939 in US. The constraints are deposited based on the recommended dietary allowances at that time. Because the simplex algorithm hadnt been established then, he had no choice but to use some complicated heuristic methods to find a solution. He began with eliminating 62 kinds of foods from the original 77 according to their lack of needed nutrients. Then he was able to estimate the amounts of each type remained in the list to be taken to reach a minimum cost by calculation. The estimate was 39.93 dollars which is merely 24 cents greater than the real optimum calculated using George Dantzigs Simplex algorithm.

Since then, linear optimization has been utilized to solve various types of diet problems with different target groups or objectives. These includes Dr. George Dantzigs similar diet problem about choosing types of food to eat on a daily basis to maximize ones feeling of feeling full with the recommended daily nutrition intake as the lower bounds of the constraints among over 500 types of food. He interpreted this objective by calculating the difference between a foods weight and the weight of its water content for each unit of foods that should be chosen according to the constraints and concluded with a minimum total. After receiving several ridiculous results, he discovered the solution to add upper bounds to the constraints for vinegar and bran in-taken and found the optimal solution using simplex method.

Specific Diet Problem

This paper was aimed to be directed only for lunch because University students and college students (19-23) find it difficult to have lunch cooked at home due to time constraint where time is essential in a competitive society. Students usually spend on fast-food restaurant because it is accessible and less time consuming.

In other words, this paper discusses an alternate way for college students to have a healthier lunch plan. The aim is to find out how a normal person of age 19 to 23 should choose lunch plan (including a main dish, a side dish, and a drink) from chosen fast food restaurants so that the weighted total of some in-take nutrients including excess calorie, sodium, cholesterol, trans fat, and total fat is minimized within 14 days period, while other in-take nutrients satisfy the Daily Recommended Intake (DRI). The chosen restaurants in this paper are Taco Bell, Subway, Pizza Hut, Tim Hortons, McDonalds, and KFC.

Model Assumptions

The following assumptions are introduced:

1) Each meal strictily consists of 1 main, 1 side, and 1 drink.

2) No perfect repetition is allowed for main+side combination within 14 days period.

3) Same restaurant can not be chosen 2 days in a row.

4) Meal can only be purchased at one place on each day.

5) Vitamins are not examined because of the lack of tyhem in fast food.

6) Breakfast and dinner are assumed to be optimally chosen. Ratio 10:6:5 is used to calculate the lunch recommended intake.

Decision Variables

First of all, we introduce the set of binary variables $\{x_{di}\}$, which represents the choice of food for each day:

$$\forall d \in \{1, 2, ..., 14\}, i \in \{1, 2, ..., 647\} :$$

$$x_{di} = \begin{cases} 1, & \text{if product } i \text{ is chosen on day } d \\ 0, & \text{otherwise} \end{cases}$$

Next, we introduce the set of binary variables $\{y_{dr}\}$, which represents the choice of restaurant for each day:

$$\forall d \in \{1,2,...,14\}, r \in \{1,2,...,6\} :$$

$$y_{dr} = \begin{cases} 1, & \text{if restaurant } r \text{ is chosen on day } d \\ 0, & \text{otherwise} \end{cases}$$

Next, we introduce the set of dependent variables $\{m_d\}, \{s_d\}, \{d_d\}$, representing the ID of main, side and drink, respectively, for each day:

$$m_d = \sum_{r=1}^{6} \sum_{i \in I_{r1}} x_{di} * i, \forall d \in \{1,2,...,14\}$$

$$s_d = \sum_{r=1}^{6} \sum_{i \in I_{r2}} x_{di} * i, \forall d \in \{1,2,...,14\}$$

$$d_d = \sum_{r=1}^{6} \sum_{i \in I_{r3}} x_{di} * i, \forall d \in \{1,2,...,14\}$$

Next, we introduce another convenient set of dependent variables $\{l_d\}$, representing the sum of ID's of main and side dish (we make use of it later when tracking repetitions):

$$l_d = m_d + s_d, \forall d \in \{1,2,...,14\}$$

The following set of binary variables $\{z_{d_1,d_2}\}$ is introduced to track repetitions in meals:

$$\forall d_1, d_2 \in \{1,2,...,14\} : z_{d_1,d_2} = \begin{cases} 1, & \text{if } l_{d_i} < l_{d_j} \\ 0, & \text{otherwise} \end{cases}$$

Sets of decision variables $\{ce_{d,1}\}$ and $\{ce_{d,2}\}$, and $\{b_{d,2}\}$ are introduced to control excess of calories for each day:

$$ce_{d,1} - \text{\# of calories consumed on day } d \text{ that is} \leq 771.43$$

$$ce_{d,2} - \text{excess, i.e. \# of calories consumed on day } d \text{ that is} > 771.43$$

$$\forall d \in \{1, 2, ..., 14\} : b_{d,2} = \begin{cases} 1, & \text{if } ce_{d,2} > 0 \\ 0, & \text{otherwise} \end{cases}$$

Model Parameters

Index r represents restaurant, index i represents food ID, index f represents food type (1 - main, 2 - side, 3 - drink). N is set of indices of all available products (items). $I_{r,f}$ represents set of indices of products in restaurant r with food type f.

$$N = \cup I_{r,f}, r \in \{1, 2, ...6\}, f \in \{1, 2, 3\}$$

$$I_{r_1,f_1} \cap I_{r_2,f_2}, \text{ if } r_1 \neq r_2, f_1 \neq f_2$$

For each product, the following parameters represent nutritions:

$$cal_i - \text{calories in product } i, i \in N$$

$$tf_i - \text{total fat in product } i, i \in N$$

$$sf_i - \text{saturated fat in product } i, i \in N$$

$$trf_i - \text{trans fat in product } i, i \in N$$

$$ch_i - \text{cholesterol in product } i, i \in N$$

$$so_i - \text{sodium inr product } i, i \in N$$

$$car_i - \text{carbohydrates in product } i, i \in N$$

$$f_i - \text{fiber in product } i, i \in N$$

$$su_i - \text{sugar in product } i, i \in N$$

$$pr_i - \text{protein in product } i, i \in N$$

Objective Function

Minimize excess of calories and total consumption of harmful nutrients: transfats, saturated fats, sodium and cholesterol. That is:

Minimize $p_1 * \sum_{d=1}^{14} \sum_{i=1}^{647} x_{d,i} * ce_{d,2} + p_2 * \sum_{d=1}^{14} \sum_{i=1}^{647} x_{d,i} * trf_i + p_3 * \sum_{d=1}^{14} \sum_{i=1}^{647} x_{d,i} * sf_i + p_4 * \sum_{d=1}^{14} \sum_{i=1}^{647} x_{d,i} * so_i + p_5 * \sum_{d=1}^{14} \sum_{i=1}^{647} x_{d,i} * ch_i$

where p_i stands for heuristically derived penalty: $p_1 = 0.01, p_2 = 1.58, p_3 = 0.06, p_4 = 0.00458, p_5 = 0.01$.

Constraints

Group 1. Binary and non-negativity constraints:

$$x_{di} = \text{bin}, y_{dr} = \text{bin}, z_{ij} = \text{bin}, b_{d,2} = \text{bin}, ce_{d,1} \geq 0, ce_{d,2} \geq 0$$

Group 2. Only one restaurant is chosen every day:

$$\sum_{r=1}^{6} y_{d,r} = 1, \forall d \in \{1, 2, ..., 14\}$$

Group 3. No restaurant is chosen 2 days in a row:

$$x_{d,r} + x_{d+1,r} \leq 1, \forall d \in \{1, 2, ..., 13\}, r \in \{1, 2, ..., 6\}$$

Group 4. Exactly 1 main, 1 side and 1 drink are purchased in the chosen restaurant every day:

$$\sum_{i \in I_{r,f}} = y_{dr}, \forall d \in \{1, 2, ..., 14\}, r \in \{1, 2, ..., 6\}, f \in \{1, 2, 3\}$$

Group 5. No perfect repetition main+side is allowed:

$$(l_{d_1} - l_{d_2}) + \mu z_{d_1,d_2} \geq 0.01, \forall d_1, d_2 \in \{1, 2, ..., 14\} : d_1 < d_2$$

$$-(l_{d_1} - l_{d_2}) + \mu(1 - z_{d_1,d_2}) \geq 0.01, \text{where } \mu = 1000000.$$

Group 6. Lunch recommend intake requirements:

$$\text{Sugar} : \sum_{i=1}^{647} x_{d,i} * su_i \leq 11.43, \forall d \in \{1, 2, ..., 14\}$$

$$\text{Calories} : \sum_{i=1}^{647} x_{d,i} * cal_i \geq 571.43, \forall d \in \{1, 2, ..., 14\}$$

$$\text{Carbohydrates} : \sum_{i=1}^{647} x_{d,i} * car_i \geq 37.14, \forall d \in \{1, 2, ..., 14\}$$

$$\text{Fibre} : \sum_{i=1}^{647} x_{d,i} * f_i \geq 10.86, \forall d \in \{1, 2, ..., 14\}$$

$$\text{Protein} : \sum_{i=1}^{647} x_{d,i} * pr_i \geq 16, \forall d \in \{1, 2, ..., 14\}$$

Group 7. This group of constraints controls the excess of calories:

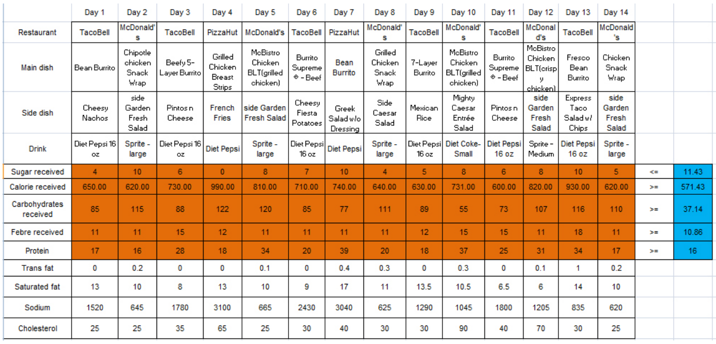
$$ce_{d,1} + ce_{d,2} = \sum_{i=1}^{647} x_{di} * cal_i, \forall d \in \{1, 2, ..., 14\}$$

$$771.43 * b_{d,2} \leq ce_{d,1} \leq 771.43, \forall d \in \{1, 2, ..., 14\}$$

$$0 \leq ce_{d,2} \leq \mu * b_{d,2}, \forall d \in \{1, 2, ..., 14\}$$

Study Results

In total, our LP model connsists of 9289 variables and 1570 constraints.



	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Day 11	Day 12	Day 13	Day 14		
Restaurant	TacoBell	McDonald's	TacoBell	PizzaHut	McDonald's	TacoBell	PizzaHut	McDonald's	TacoBell	McDonald's	TacoBell	McDonald's	TacoBell	McDonald's		
Main dish	Bean Burrito	Chipotle chicken Snack Wrap	Beefy 5-Layer Burrito	Grilled Chicken Breast Strips	McBistro Chicken BLT(grilled chicken)	Burrito Supreme ® - Beef	Bean Burrito	Grilled Chicken Snack Wrap	7-Layer Burrito	McBistro Chicken BLT(grilled chicken)	Burrito Supreme ® - Beef	McBistro Chicken BLT(crispy chicken)	Fresco Bean Burrito	Chicken Snack Wrap		
Side dish	Cheesy Nachos	side Garden Fresh Salad	Pintos n Cheese	French Fries	side Garden Fresh Salad	Cheesy Fiesta Potatoes	Greek Salad w/o Dressing	Side Caesar Salad	Mexican Rice	Mighty Caesar Entrée Salad	Pintos n Cheese	side Garden Fresh Salad	Express Taco Salad w/ Chips	side Garden Fresh Salad		
Drink	Diet Pepsi 16 oz	Sprite - large	Diet Pepsi 16 oz	Diet Pepsi	Sprite - large	Diet Pepsi 16 oz	Diet Pepsi	Sprite - large	Diet Pepsi 16 oz	Diet Coke-Small	Diet Pepsi 16 oz	Sprite - Medium	Diet Pepsi 16 oz	Sprite - large		
Sugar received	4	10	6	0	8	7	10	4	5	8	6	8	10	5	<=	11.43
Calorie received	650.00	620.00	730.00	990.00	810.00	710.00	740.00	640.00	630.00	731.00	600.00	820.00	930.00	620.00	>=	571.43
Carbohydrates received	85	115	88	122	120	85	77	111	89	55	73	107	116	110	>=	37.14
Febre received	11	11	15	12	11	11	11	11	12	15	15	11	18	11	>=	10.86
Protein	17	16	28	18	34	20	39	20	18	37	25	31	34	17	>=	16
Trans fat	0	0.2	0	0	0.1	0	0.4	0.3	0	0.3	0	0.1	1	0.2		
Saturated fat	13	10	8	13	10	9	17	11	13.5	10.5	6.5	6	14	10		
Sodium	1520	645	1780	3100	665	2430	3040	625	1290	1045	1800	1205	835	620		
Cholesterol	25	25	35	65	25	30	40	30	30	90	40	70	30	25		

Figure 1: 2 weeks lunch menu plan

Using OpenSolver software, the optimal solution was obtained. The following table provides detailed lunch plan, including choice of restaurant, choice of food, and nutrition contents:

Interpretation

For the interpretation of the result, Excel open solver has produced an optimal solution, minimizing cholesterol to 540mg, sodium to 15,270mg, trans-fat to 1.7mg, saturated fat to 161mg, and calories to 10,161cal. Out of these 6 fast-food restaurants, the following 3 restaurants were chosen for lunch; Macdonalds, Taco Bell, and Pizza Hut. Among all of the drinks for the 14 days of lunch, Diet Coke and Sprite have been chosen for the drinks. No burgers were chosen for the menu plan, most of the menu items were sandwiches and fresh salads. The other 3 restaurants; KFC, Subway, Tim Hortons, were not accepted to be in the final output. KFC had items that does not satisfy either the constraints or cannot find a healthy solution that minimizes the objective function. Tim Hortons and Subway might be included depending on the data they have provided, which didnt include any soups and had limited amount of side dishes cookies, chips, and donuts that did not satisfy the constraints or a solution that minimizes our objective function.

Although our model is extremely flexible, further research and improvement can be made. One of the areas for such improvement is adding option for having meal without side (for example, to accomodate special restaurants), as well as option for buying parts of meal in two distinct places (to accomodate food corts). Increasing number of restaraunts and available products will give more various and, possibly, more healthy meal.

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