RECIPROCAL INFLUENCES IN A DUO OF ARTEFACTS: IDENTIFICATION OF RELATIONSHIPS THAT SERVES TO MULTIPLICATIVE THINKING

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Abstract

The combined use of a physical pedagogical artefact and its digital counterpart is described as a duo of artefact. In the literature, duos of artefacts are mostly presented with a certain order: non-digital artefact is followed by the digital counterpart. This study examines the influence of reciprocal use of artefacts in a duo on a 5-year-old child's identification of multiplicative relationships between the objects. Data is created through the video record of two clinical interviews with the child. The results showed that the reciprocal use of the artefacts enriched the child's experiences of each artefact and mediated the relationships which were important for multiplicative thinking.

Keywords: duo of artefacts, multiplicative thinking, drawing, educational technology

Introduction

Studies show that mathematical tasks which require students to manipulate physical artefacts enhance mathematical teaching and learning (Carbonneau et al., 2013). However, the rigid structure of artefacts might prevent teacher from modifying them in a way to increase their mathematical potentials. At this point, their digital counterparts add value to the use of physical objects as classroom teaching equipment because different artefacts trigger different signs (e.g. natural language, gestures, and mathematical semiotic systems), and different signs lead to different meanings. Digital counterpart can achieve this through “offering students a new opportunity to identify the mathematical properties embedded in the artefact behavior and more abstract and conventional representation of mathematical objects” (Soury-Lavergne, 2017, p.1). This combined use of a physical pedagogical artefact and its digital counterpart is described as *duo of artefacts* (Maschietto & Soury-Lavergne, 2013).

Integrating duo of artefacts in mathematics classes is a recent practice, but it has already demonstrated some positive outcomes (see below for more detail). In most of these studies, the duo of artefacts is presented with a certain order: first, students are introduced the physical artefact and then they are given the digital counterpart. This restrictive order suggests that the duo of artefacts enhances mathematical ideas through the added value of digital counterpart only. However, this one-directional approach might hinder the potential of physical artefact to enrich the affordances of the digital counterpart. In the literature, various duos of artefacts have been used to introduce students to various mathematical topics. I will study how reciprocal use of a duo of artefacts enhances the mathematical ideas related to multiplicative thinking which, to the author’s knowledge, has not been studied with respect to a duo of artefacts yet. In this study, the digital artefact is a free tablet application called TouchTimes (Jackiw & Sinclair, 2019) which is designed to develop multiplicative thinking through creating quantities in specific ways. The physical artefact is the pencil and paper, through which students draw the target numbers they created with Zaplify – one of the TouchTimes “worlds”.

Duo of Artefacts

Drawing on instrumental approach, Soury-Lavergne (2021) proposes a difference between “two artefacts” and “duo of artefacts”. According to the instrumental approach, when individuals encounter an artefact (material entity), they construct utilization schemes (psychological entity) as they interact with the artefact. The combination of the material and the psychological entities generates a specific instrument for the individual. This is called instrumental genesis. For example, upon seeing a plastic circular object (material entity), someone might think of placing it on the paper and circumscribing (psychological entity) to create a geometrical diagram.

The difference between “two artefacts” and “duo of artefacts” depends on the nature of instrumental genesis they prompt. The former suggests two separate instrumental geneses of two separate artefacts. Whereas the duo of artefacts constitutes a system that emerges through the joint instrumental genesis of two artefacts. Soury-Lavergne (2021) acknowledges that the new instruments integrate the previously developed instruments into its form creating a system rather than an isolated independent instrument. As it is not practical to identify all the previous instruments in a system, she proposes to reduce the complex system of instruments into duo of a tangible entity and a digital one to study their influence on learning.

Drawing on Bourmaud, Soury-Lavergne (2021) indicates three conditions for the joint instrumental genesis triggered by a duo of artefact: complementarity, continuity and antagonism. When two artefacts are used together (either simultaneously or successively) they complement each other. However, the complementary use of artefacts may not result in a joint instrumental genesis without a continuity between them. When the artefacts are used in relation to each other, shared characteristics or elements of the artefacts build a continuity. On the other hand, the divergent features/functionalities of the artefacts result in antagonism between them. These divergences create constraints for the users' existing schemes and prompt them to adapt their schemes when passing from one artefact to the other.

These three conditions explain why providing two artefacts may not be effective in creating a system of artefacts that results in joint instrumental genesis. This is illustrated in Lei et al. (2018) that examined an ineffective combination of a material and a digital tool. The material artefacts were a tape measure and theodolite. Whereas the digital artefacts were two apps installed in tablets called EasyMeasure and Angle Meter. The teacher provided the students with this duo of artefacts to introduce the concept of percentage error. One of the main reasons Lei et al. (2018) attributed to the failure of the duo was the difference between the artefacts. Apart from their functions, which was to measure, they did not share any feature. When we consider Lei et al.'s (2018) finding with respect to the conditions cited by Soury-Lavergne (2021), it could be said that there is little opportunity not only for continuity but also for antagonism. So, unlike a duo of artefact, these two tools did not lead to a system of artefacts that triggered a joint instrumental genesis. Therefore, it is not appropriate to call them as duo of artefacts from Soury-Lavergne's perspective. Unlike this counterexample, the literature presents various successful use of duo of artefacts in teaching and learning mathematic. The following section summarizes a few of them. The exemplar studies are chosen to represent the diverse use of duos in mathematics lessons.

Teaching and learning through various duos of artefacts

Maschietto (2018) studied how the Pythagorean theorem was introduced to 7-grade students in a composite environment which consisted of a material and a digital tool. One of the material tools was a mathematical machine which consisted of four congruent wooden right triangles that fitted into a wooden square. The square was covered with a red paper and surrounded by a frame. The digital tool was an Interactive Whiteboard (IWB) on which the teacher created the digital version of the mathematical machine. The tasks were (1) to obtain red square areas by placing the triangle prisms into the square frame and (2) to change the configuration to obtain a larger red square which is surrounded by the triangle prisms. While students directly manipulated the mathematical machine, the digital tool was manipulated only by the teacher and a few students to switch between the configurations of the triangles on the board. Even though many students did not manipulate the digital tool directly, Maschietto (2018) proposed that the conservation of the square areas was emphasized through linking the manipulations of the triangles in the digital tool with the manipulations of the triangles in the mathematical machine. This conversation helped students deduce the Pythagorean theorem.

Van Bommel and Palmér (2018) compared six-year-old students' responses to a combinatorial task when they used only physical artefacts and when they used a duo of artefacts. The task was to find how many different ways three toy bears can be arranged in a row on a sofa. The physical artefacts were the toy bears, paper and a number of coloured pencils to record the arrangements. The analysis of the children's drawings revealed many duplicates in students' solutions and thus indicated that students did not systematize their solutions. The digital artefact was designed based on these findings to provide the children with feedback about the duplicates. When the students used the duo of artefacts, they were first introduced the digital artefact and then asked to find the number of seating arrangement by using paper and pencil. The results show that the children who solved the task via the duo of artefacts were found to keep more systematic records of the situations and to enhance their understanding of what a duplicate means in a combinatorial problem.

Soury-Lavergne and Maschietto (2015) studied how a duo of artefacts was used by six years-old students to learn about numbers. The students first worked with pascaline, a mechanical machine made of gears which allowed students to create and to add numbers symbolically by rotating them. The digital counterpart of pascaline was embedded in an e-book. The students were given two tasks. One of them asked students to add two numbers. The other one asked them to write a number with minimum rotations. The findings showed that the duo of artefacts prompted the students to connect the separate conceptualizations of quantity and digit.

All the duos used in these studies conform to the three principles that would result in a joint instrumental genesis. While they differ from each other in terms of mathematical topics they develop, the type of artefacts involved in the duo and the nature of the tasks they posed; the order of the artefacts was the same across all of these studies: either the digital artifact was followed by the non-digital counterpart or vice versa except for one case. In Soury-Lavergne and Maschietto (2015), one teacher made the physical artefact available again after the students had difficulty to solve the tasks in e-pascaline. This bi-directional use of duo is unique among these studies and it suggests a new way to exploit the potential of the duo. Compared to using each element of a duo individually in successive occasions, manipulating them reciprocally during a mathematical activity might enhance the integration of instrumental geneses more strongly.

In this study, I will examine reciprocal use of a duo which involves pen and paper as its non-digital element. Compared to the artefacts like the mechanical machines used in Soury-Lavergne and Maschietto (2015) and Maschietto (2018), pen and paper provides students with a special medium to create meanings with less restrictions that stem from the physical structure of the artefact. This use of pen and paper is different from using drawings only to express and record thoughts after manipulating the mechanical artifact, which was the case in all three studies. However, the unrestricted diagramming might deviate learners from the target mathematical idea unless it is repeatedly restructured based on the manipulation of the digital artifact which embeds the intended mathematical relationships within its design in this study the multiplicative relationships.

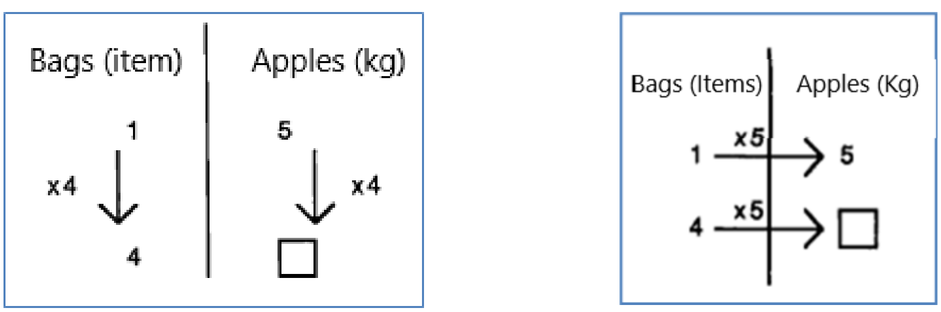
Multiplicative Thinking

Multiplicative thinking is conceptualized by many researchers in a unique way. Even though they slightly differ from each other and focus on the different aspects of the concept, one thing is shared by all: it is different from additive thinking. Schwartz (1988) focuses on the referents of quantities in these operations. While the quantities refer to the same entity in addition (e.g., 5apples + 4 apples = 9 apples), different type of quantities are operated on in a multiplication (e.g., 5 kg of apples per bag x 4 bags=15 kg apples). Similarly, Clark and Kamii, (1996) points to the abstraction of the number of units involved in both operations. While addition is conducted with only one unit-count (that quantifies only the individual apples in the previous example), in multiplication one operates on two unit-counts (one that quantifies the kilos of apples, the other that quantifies the price of the apples).

Vergnaud (1988) emphasizes the relationship between the unit counts a child establishes in an operation and distinguishes scalar relationships from functional ones. For example, when asked the problem "Amy wants to buy 3 bags of apples. Each bag has 5 kg of apples. How many kilos of apples does she buy in total?", a student might show the solution either with 4x5=20 or with 5x4=20. Even though they are both multiplications, Vergnaud says that "the relationships that leading to these choices are very different" (p. 145) and illustrates the difference using the following T tables in figure 1.

Figure 1

Illustration of 4x5=20 and 5x4=20



Note. Adapted from "Multiplicative structures" by G. Vergnaud, G., in J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 141–161), 1988, Lawrence Erlbaum Associates. Copyright 1988 by The National Council of Teachers of Mathematics.

In the first case students attends to the ratio between the same quantities which is a scalar. Therefore, 4x5 is a "concatenation" of 5+5+5+5: the amount of apples =the amount of 1 bag, plus the amount of 1 bag, plus the amount of 1 bag, plus the amount of 1 bag (Vergnaud, 1988, p. 146). Whereas in the second case (5x4) the student attends to the ratio between the different quantities. In this case 5 is not a scalar, it is associated with a many-to-one correspondence between the unit counts: 5 kilos per 1 bag.

In addition to this static relationship between the two unit-counts, Davydov (1992) points to a dynamic feature of multiplication when he defines it as the transfer of unit counts. He explains the meaning of multiplication with respect to measuring activities and distinguishes a small and a large unit-count which both quantify a given magnitude of an object. Measuring a magnitude (e.g. apples) with the small unit (kg) would be impractical. Therefore, one indirectly quantifies the magnitude in relation to the smaller unit by transferring the unit count from the smaller to the larger (bags) thanks to the established relationship between the two (5kg/bag). This transfer implies a simultaneous multiplicative action.

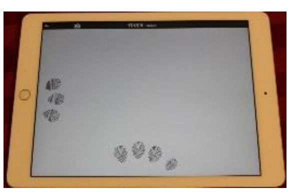
Drawing on Davydov's notion of transfer of unit-count and Vergnaud's notion of functional relationship, Jackiw and Sinclair (2019) designed TouchTimes (*TT*) to enhance multiplicative thinking. *TT* consists of two models or "worlds" – Zaplify and Grasplify. Davydov's and Vergnaud's multiplicative notions are conveyed in both worlds, yet through distinct models. Thus, Zaplify and Grasplify prompt learners to experience these multiplicative ideas in two different ways. This paper will focus only on the former world (see Bakos & Pimm, 2020) for more details on how Grasplify world prompts these multiplicative notions).

Zaplify

When entered, this world shows an empty screen. When the tablet is placed horizontally on a surface, seven fingerprints and a diagonal line appear respectively in order to guide users to place their fingers both horizontally and vertically in the designated areas separated by the diagonal (see Figure 2a &2b).

Figure 2

(a) Fingerprints and (b) fingerprints and the diagonal.

a) 

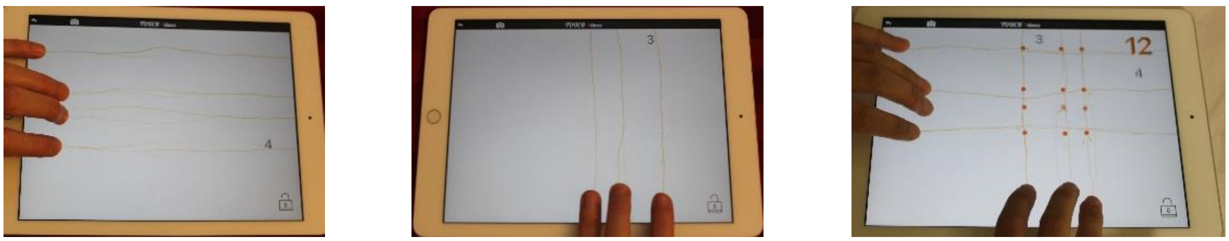
b) 

When a user places and holds any finger on the screen, a "lightening rod" (I will call them "lines" from now on), which passes through the point of touch and crackles dynamically, appears on the screen either horizontally or vertically according to the position of the touch with respect to the diagonal. The upper-left triangular area formed by the diagonal allows horizontal lines (HL), while the lower-right triangular area allows vertical lines (VL). Screen contact can be made with one finger at a time or with multiple fingers simultaneously. Multiple fingers that maintain continuous contact can create either only HL, only VL or both VLs and HLs (see Figure 3 a-c).

Whenever two perpendicular lines intersect, an orange disc gradually appears on the intersection points. The numerical value of the total number of intersections, which is the product of the two factors, appears in the upper right corner of the screen (see Figure 3c). If there is no intersection, only the number of factors appear (see Figure 3 a,b).

Figure 3

(a)HLs, *(b) VLs* *(c) VLs and HLs*



There are two modes of manipulation of the app: locked and unlocked. In the unlocked mode, the lines disappear as the fingers separate from the screen, whereas in the locked mode, lines remain on the screen even when the user's finger is lifted, but no longer crackle dynamically. This allows a user to create products that involve more than ten fingers.

The Zaplify objects, the gestures that create these objects and the relationship between these objects are all associated with various aspects of multiplicative thinking. The vertical and the horizontal lines represent the two unit-counts of multiplication. The orientations of lines may help students distinguish these units-counts. In addition to this visual difference, the separation of the units may be associated with the difference in the haptic experiences. While the horizontal lines can be created only by touching the upper triangular area, one must touch the lower triangular area to create vertical lines. Pressing fingers to create parallel lines on one triangular area and then pressing down a finger on the opposite side to create a perpendicular line canbe associated with Davydov's notion of transfer of unit counts. In this case, the unit count is transferred from the parallel lines to the perpendicular line. This transfer results in Vergnaud's notion of many-to-one correspondence between the units: many units represented by the parallel lines correspond to the new unit which is represented by the perpendicular line (see Güneş, 2021, for a more detailed explanation of how Zaplify can prompt multiplicative thinking).

Theoretical Framework

This study draws on Bartolini Bussi and Mariotti's Theory of Semiotic Mediation (TSM). This theory focuses on the relationship between the representation systems and the human cognition. Human beings create representations through using artefacts and this has two consequences: the modification of the environment and the cognitive development. TSM is based on this double nature of artefacts.

An artefact does not guarantee a specific use for the subject. Indeed, Rabardel (as cited in Bussi & Mariotti, 2008) distinguishes artefacts from instruments. An artefact is a concrete or a symbolic object itself. It becomes an instrument by the subject through its particular use. For example, a glass is an object which is designed to carry liquid. If a cook uses it to crash some walnuts into smaller pieces by pressing the walnuts between the bottom of the glass and a cutting plate, the glass becomes an instrument.

The instrumental approach to artefacts can be informative in analyzing the cognitive processes related to the use of a specific artefact and its semiotic potential. However, it is not adequate to analyze the more complex process of teaching and learning mathematics through artefact use. At this point, Bartolini Bussi and Mariotti (2008) resort to Vygotsky's approach to artefacts.

Vygotsky talks about the difference between an individual's developmental levels in two different situations: (1) when an individual is able to accomplish a task him/herself, and (2) when an individual can accomplish a task with the guidance of a more knowledgeable individual (as cited in Bussi & Mariotti, 2008). This difference is called the *zone of proximal development*. Within this zone, the communication between the individual and the more knowledgeable one leads to the cognitive development of the learner. The theory of semiotic mediation elaborates more on the relationship between tasks, signs and mathematical meaning making within this process and distinguishes semiotic mediation of artefacts from teachers' cultural mediation.

Using an artefact in a social context, learners produce certain signs which are essential for semiotic mediation. These signs have a dual role: expressing the relationship between the task and the artefact on the one hand, and the relationship between the artefact and mathematical meaning on the other hand. The former is called an *artefact sign* and their meaning is associated with the operations conducted to achieve the task. The latter is called a *mathematical sign* and it is aligned with the existing mathematical culture. On the way to the evolution of artefact signs into mathematical signs, pivot signs are important. The pivot signs "may refer both to the activity with the artefact...and to the mathematical domain" and they are distinguished from the other signs based on the extent of generalization they carry (Bussi & Mariotti, 2008, p.757). In this study I asked how the signs evolved during reciprocal use of a duo of artefact.

Method

Data is created through the video recording of two clinical interviews with a 5-year-old child, whom I name Zach. Both interviews lasted for approximately half an hour. Zach used Zaplify and pencil-and-paper during the interviews. The interviews consisted of number-making tasks, drawing tasks, and what-happens task in which I (denoted as R in the below transcripts) asked Zach (denoted as Z in the below transcripts) to anticipate how the number would change if I added more fingers.

Clinical interviews conducted in this study could be described as the derivative of joint inquiry activities which naturally occur in every individual's life (DiSessa, 2007). I conducted the interviews at Zach's home. Zach's father (denoted as F in the below transcripts) was present during the first interview, and he participated in the interview by asking questions to Zach when he seemed hesitant to respond. My goal was to help Zach to make sense of Zaplify and to discover how he makes sense of it. Even though the interviews did not carry an instructional orientation (I avoided evaluative comments based on a normative response to the tasks), it would be problematic to deny that manipulating the artefacts while communicating with the interviewer did not contribute to Zach's learning.

The participant is recruited through convenience sampling. Multiplication is generally introduced in the second and the third grade of elementary schools. However, studies show that before formal schooling, young children can demonstrate some aspects of multiplicative thinking (Bakker et al., 2014), for example by extracting the invariant proportional relationship between two numerical magnitudes (McCrink & Spelke, 2010). Therefore, choosing a young participant, this study also contributes to the discussion of whether multiplicative thinking can be developed with instruction in younger ages (as per Askew, 2018) and whether the ordering of the mathematical topics in the curriculum documents that positions learning of multiplication after addition based on a hypothesized developmental learning progressions can be challenged (as per Bicknell, et al., 2016).

In this analysis, I focused on the signs Zach created via the duo of artefacts, drawing from Arzarello et al.'s (2009) concept of *semiotic bundle*. There are two ways to analyze a semiotic bundle: synchronic and diachronic analysis. The former focuses on a specific moment where the subject produces different signs spontaneously. The latter focuses on the evolution of the signs produced by the subject in successive moments. I also analyzed different signs created by different artefacts at different time points in a synchronic manner in order to examine the relationship between the artefact signs.

Findings

In the following, I highlight how Zach identified relationship between mathematical objects via duo of artefacts. I characterize the instances with excerpts from the interviews.

At the beginning of the first interview session, Zach randomly made one orange disc on Zaplify. Zach described the orange disc as a dot. When I asked him to make one more, he could not make it. During the following 18 minutes, while Zach was holding HLs, I was adding VLs one by one, making 2, 4, 6, 8, 10, and 3, 6, 9, 12, 15 respectively. Then I asked Zach to make "one" again, assuming that creating numbers repeatedly on Zaplify might have helped Zach to identify the relationship between the lines and the discs. As I pointed to the upper right corner of the screen, I said: "I want to see the [numeral] one here and one orange ball". After a few attempts, he could not make any disc. Then I asked him to draw one disc:

1 R: In order to get one dot, what we should see? How does one dot appear? Can you draw one dot? How was it on the screen when we see one dot?

2 Z: It was small and red [*drawing a circle*]

3 R: Were there anything else other than the dot?

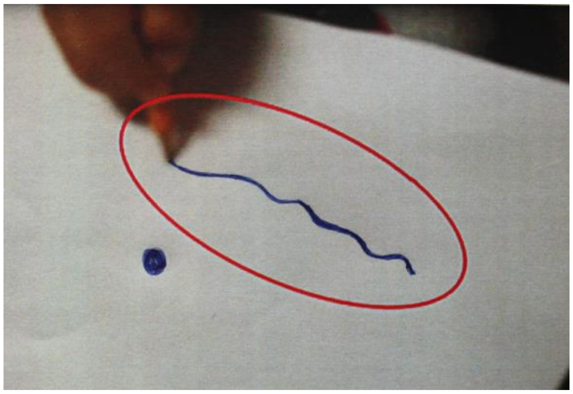
4 Z: A yellow line

5 R: Where was it?

6 Z: … [drawing a curvy line which looks like a wave just below the circle]

Figure 4

Horizontal curly line



Note. The author retraced the pencil marks in the pictures to improve visibility.

Zach used the words "small" and "red" in order to describe the dot. These artefact signs refer to physical features of the ball unlike its position, which might suggest a relationship between the other artefact signs such as lines and the intersection point. When I drew Zach's attention to the other artefact signs (line 3), Zach uttered the word "yellow line". This artefact sign includes a mathematical sign, which is a "line", yet it also refers to the color of the line in order to describe it. Again Zach created signs related to the physical features of the objects rather than their orientation (e.g. horizontal/vertical), which is important in terms of multiplicative relationships. When I hinted the orientation by asking where it was (line 5), Zach created a sign in another modality. Rather than describing it with verbal signs, he created a visual sign with his drawing (see Figure 4). This sign illustrates the line in horizontal orientation as in the Zaplify, yet separate from the disc. So it seems that Zach did not relate the disc with the HL except for their quantities. For one disc, he created one line.

The relationship between the signs appeared in our second trial. After Zach and I together made a disc the second time on Zaplify, I asked him to draw a disc on the paper.

7 R: How did we do one dot? Can you draw it?

8 Z: … [drawing a circle]

9 F: Draw what you saw on the screen. Where were the yellow lines?

10 Z: Where were the yellow lines? One is here and one is here.

11 R: Why don't you draw it here [*pointing to the paper*]

12 Z: … [drawing one vertical curly line from top to the bottom of the paper, then another one from left to right of the paper crossing over the VL]

13 F: [*pointing to the dot on the paper*] Is this dot on the same spot compared to the screen?

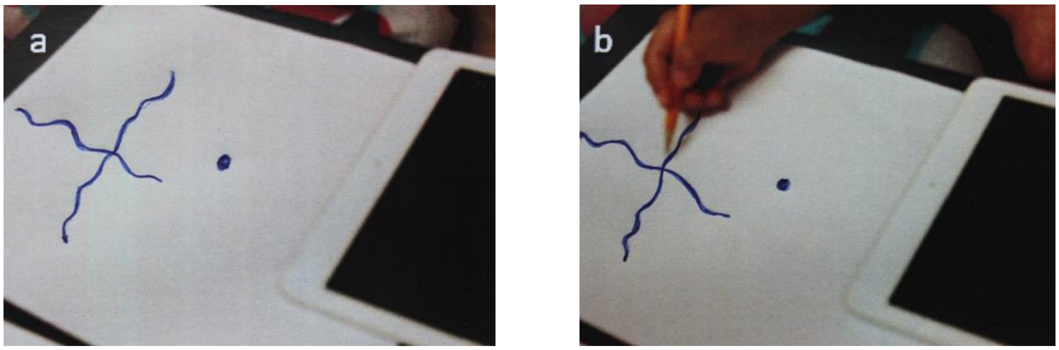
14 Z: No.

15 F: Draw the dot. Where should it be?

16 Z: It should be in the middle of here [*pointing the intersection of the lines*]

Figure 5

(a) Dots and the intersecting lines, (b) pointing to the intersection of the lines



Note. The author retraced the pencil marks in the pictures to improve visibility.

Compared to the first drawing, Zach produced more signs in this episode. First, he drew one disc and then two lines next to the disc, which intersected each other. So this physical separation between the lines and the disc in Zach's drawing indicates partial relationship between the artefact signs in that the lines are related to each other, but they are not necessarily related to the disc.

Zach transferred the orientation of the lines from Zaplify to the paper directly. He drew two perpendicular lines as in Zaplify (see Figure 5a). When we made one disc together, Zach first held his finger and made a VL, and then I put my finger and made a HL. Similarly, first he drew the VL in this episode. While the order of the lines created in Zaplify was mirroredin his drawing, it was not the case for the order of the disc. In Zaplify, the disc appeared following the lines, but on the paper, he first drew the disc and then the lines. Thus, he did not transfer the location of the disc in relation to the lines in his darwing. Zach connected the disc with the lines (see Figure 5b) only after he was asked to compare his drawing of the disc with the diagram in the Zaplify (no. 13-16).

Zach started to create the intersecting lines on the screen after he used his second drawing as a reference to make one disc in Zaplify. However, the relationship between the intersecting points and the discs became solid after we discussed the relationship between the lines at the second interview. Until this episode, Zach answered few "what happens" tasks correctly. After our discussion, he started to demonstrate a consistent strategy to answer these tasks correctly. The following episode presents one such discussion:

After Zach made one disc on the screen, I asked him: "What happens here?" as I pointed to the intersection of the lines.

17 Z: One dot.

18 R: What is happening to the lines here where the dot stays [*pointing to the intersection*]?

19 Z: The dot stays in the middle [*pointing to the dot*] of these [*tracing the VLs and the HLs*] lines

20 R: How did you make this [*pointing to the dot*] in the middle?

21 Z: I put my finger here [*pointing to the bottom of the VL*] and make the line, and then I put my finger here [*pointing the HL*] and make the line, and then I make the dot with this line [*tracing the HL back and forth*]

22 R: You made this line [*pointing the VL*] first, and this one [*pointing the HL*] second, right?

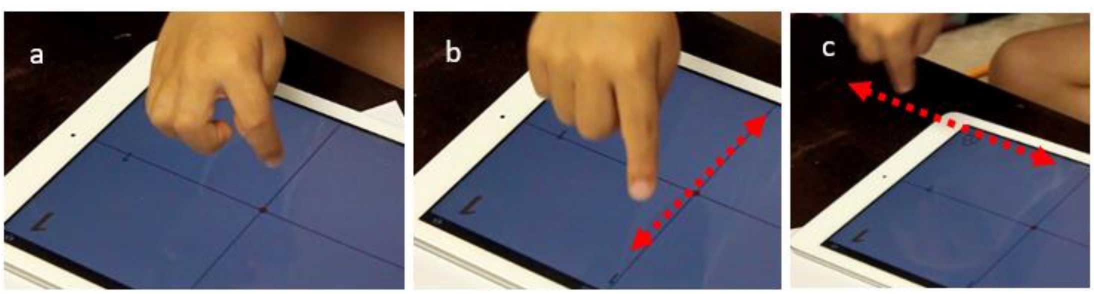
23 Z: Yes.

24 R: What did the second line do to the first line? What happened here [*pointing the intersection*]?

25 Z: Second line crossed [*tracing the HL*] the first line [*tracing the VL*]. The dot is with the second line.

Figure 6

(a) Pointing to the dot, (b) Tracing the VL, (c) Tracing the HL



Zach referred to the intersection point via a sign "the middle", which he created during a drawing task in the previous interview (line 16). The verbal sign "the middle" and "these lines" are used together with gestures (line 19). They all together suggest that the orientation and the intersection point of the lines are both related to the location of the disc. The pointing gesture (see Figure 6a) and the word "middle" refer to the intersection point, and the tracing of the lines (see Figure 6b & 6c) refers to the perpendicular lines. According to Zach's verbal accounts, the intersection seems to be necessary for the disc to appear. He stated that he made the disc with the second line, which crossed the first line (line 25). Thus, the sign "cross" points to the relationship between the lines and it is an important sign to create the disc.

Discussion and Conclusion

In this study I examined the evolution of signs during the reciprocal use of a duo of artefact. The digital artefact was Zaplify which was an iPad application designed to develop multiplicative thinking. The non-digital artefact was pen and paper. The tasks were designed for the duo to help a five-year-old child to identify relationships which can be associated with the two unit counts of multiplication (as per Clark & Kamii, 1996, Davydov, 1992, Schwartz, 1988, and Vergnaud, 1988) and the functional relationship between them (as per Davydov, 1992, and Vergnaud, 1988). So rather than to multiply two numbers correctly, the child was prompted to sense multiplicative notions by distinguishing HL's and VL's of Zaplify which represent two factors of multiplication and by making one object (the dot) out of two objects (the lines), which is contradictory to additive thinking.

The findings show that after manipulating the digital artefact, the child first created the signs which were related to the individual characteristics of the objects such as their shape (e.g., curly lines), their size (e.g., small dot), and their colors (e.g., yellow line), instead of the spatial relationship between the objects. Moreover, the former signs illustrated more additive thinking. The child created one line next to the dot when asked to make the numeral 1. This might indicate that for the child the numeral which symbolizes the dot must be created with one object which is the single line. By interacting reciprocally with each element of the duo, the child started to create signs which expressed the spatial relationships among the Zaplify objects and to create quantities in a way which would challenge the additive relationships between the objects.

The result of this study shows that a child as young as five years old can fluently identify the difference between the referents of the quantities and coordinate them to create a multiplicative product after interacting with a duo of artefact which is designed to prompt multiplicative thinking. Thus, it supports Askew's (2018) finding that under the appropriate instruction younger children can also learn multiplicative concepts which are assumed to be difficult for them. Even though the child might have been introduced some notions related to addition in the kindergarden or by his family, he has not been formally trained on addition which happens in the grade 1. Therefore, like Bicknell et al., (2016), this study also challenges the hypothetical learning trajectory which situates learning of multiplication after the formal introduction of addition.

The findings show that creating dots in Zaplify was not enough for the child to right away identify the multiplicative relationships between the objects. At the beginning of the interview, while exploring the app, Zach created a dot right away probably by chance as he could not achieve it when the interviewer asked him to make a dot again. Then he made many dots with the interviewer for a relatively long time (18 minutes). He started to express the relationships between the Zaplify objects after drawing. However, moving from manipulating the digital artefact to drawing the screen configuration in one cycle was not effective to make the relationships between the objects salient, either. Zach created several pivot signs in different modalities via reciprocal use of this duo of artefacts in several cycles before he fluently answered the "what happens" questions which required identification of the relationships between the lines and the dots.

This study does not propose that the digital artefact must be provided with the non-digital counterpart to develop multiplicative thinking. The child might have identified these multiplicative relationships after interacting only with the digital artefact for a longer time with additional tasks which prompt him to compare various configurations of his fingers with the resulting products. However, I propose that shifting between manipulating the digital artefact and drawing has a potential to speed up the process of identifying the multiplicative relationships.

In addition to accelerating learning process, the reciprocal use of duo helped the child build various meanings for the lines. As soon as a finger is pressed on the screen, a line always appears as a complete discrete object. Whereas the child created a line on paper as the trace of a continuous hand movement. However, these varying meanings attributed to the Zaplify objects were not confined to the specific medium they were created. Zach's verbal accounts that described the relationship between the Zaplify objects were accompanied with dynamic gestures that mirrored his drawings. These dynamic gestures were accompanied some verbal signs (e.g., "the second line crosses the first line") which emphasized the relationship between the lines. Mariotti and Montone (2020) describe this interaction as the synergy between the artefacts of the duo. So, the reciprocal use of the duo enriched the child's experience of multiplicative relationships embedded in the digital artefact through this synergy. In this study pen and paper provided the child with a medium to build and extend meanings in addition to record his interpretation of the digital artefact (de Freitas & Sinclair, 2012; Thouless & Gifford, 2019).

While interacting with the duo of artefacts, Zach was communicating with the adults most of the time. Therefore, discussing with adults (both the researcher and the father) through specific signs seemed to play a role in mediating the relationship between one disc and the intersection point of two lines. While these discussions guided the child to attend to specific relationships, the child's responses did not always indicate an alignment with the intended direction of the adults' questions. For example, the interviewer asked "how" questions to direct the child's attention to the process of making a dot. While the child responded to these questions by creating independent static signs (e.g., drawing of a single dot, saying "a yellow line") at the beginning of the interview, his responses included multiple signs in relation to each other (e.g., tracing gesture on both lines in Zaplify) as his interaction with the duo of artefact progressed.

This study presents the preliminary results of reciprocal use of a duo that is designed to develop multiplicative thinking. These tentative findings show that moving repeatedly back and ford between each element of the duo while communicating with others can accelerate students' meaning making process and expand their meanings by prompting a synergy between the two media. The next step will be to analyse the relationship between the signs created through each element of the duo based on extensive data.

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