EXPERIENCING LEARNING MATHEMATICS AND REFLECTION: CALCULUS 12 PARTICIPANTS' STUDY

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Abstract

This study focuses on the assessment strategy that was designed in 2017-2018 academic year in two Calculus 12 classes. Students affect was at the centre of the research questions, thus clinical interviews were used to create data on the relationship with mathematics as well as personal reflections on the learning of mathematics in the given year and overall in students' experience in school. Grounded Theory was used as a research methodology bringing the emerging themes to the surface. The analysis showed that students were interested in reflecting on their learning and the new approach to assessment made a positive change in their relationship with leaning in a mathematics classroom.

Keywords: standards-based grading, assessment, motivation, affect, mathematics, classroom

Introduction

Assessment in mathematics classrooms has been a very hot topic in the field lately, as it appears to be full of tension. There are powerful voices trying to influence the practice of teaching in classrooms and beyond, as there are decades-old discussions with unresolved problems in defining terms and explaining phenomena (Frey & Schmitt, 2007). There is a strong traditional pull of a system of tests and quizzes as historically practitioners have been exposed and graduated from such a system (Buhagiar, 2007; Romagnano, 2001). Furthermore, because of the strong traditional influence, there are instances of masking the old traditions in the innovative kind (Shepard, 2005). This is driven by the fact that assessment has taken place primarily for the purpose of evaluation (McTighe & O'Connor, 2005). The other side of the argument is calling for stepping away from the evaluative nature and aligning a new purpose of assessment: "Classroom assessment and grading practices have the potential not only to measure and report learning, but also to promote it" (McTighe & O'Connor, 2005, p. 10). One part of the argument to change assessment has been feedback, as it is claimed to be most effective function for improving student learning (Guskey & Bailey, 2001; Wiggins, 1998). The above literature influenced an attempt to change assessment in 2017-2018 Calculus 12 class taught at an independent school in Lower Mainland of British Columbia, Canada.

The Study

The main concern coming into the position of change was noticing of students' experiences in a math class. The following graphical representation was produced to illustrate these experiences based on classroom observations of students' behaviour, body language, and reaction to the results to major assessments in the previous offerings of this course:

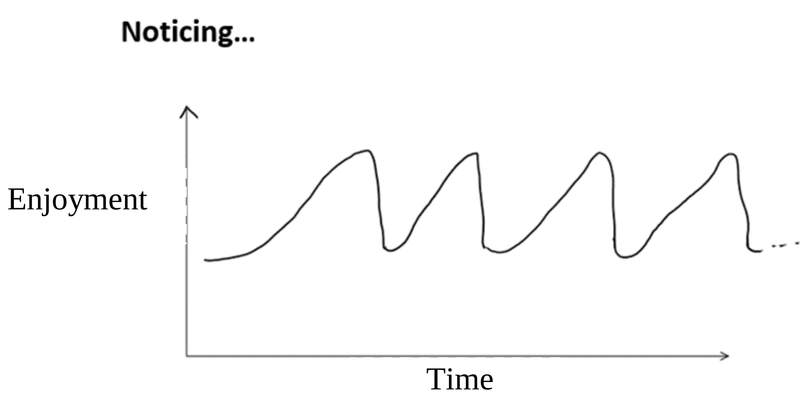

Figure 1. Noticing Illustration

In Figure 1 the horizontal axis is time and vertical axis is confidence, enjoyment, or positive feeling. The dips correspond to an assessment, usually a test given to students. This pattern, once thought about for a while, makes a scary contemplation with a simple calculation: 5 years of high school times an average of 8 tests per course = 40 such dips! If that pattern does teach learners something, it is definitely not a positive correlation with mathematics and their experience with it. With this in mind, the following assessment method, called the Check-Point System, was devised for the 2017-2018 year. The outline of the course was broken down into major topics and subtopics. Each subtopic became a trackable element for each student, which they could view at any point in time as a shared google sheet. So, instead of a regular marks book, now every student had the profile with their continuous progress, as each of the subtopics was repeatedly assessed. Another option given to students was an interview at the end of each term. In this interview students could showcase that they know a certain subtopic better than their overall mark for it. Below is the screen shot of one such spreadsheet:

Description	CP 1	CP 2	CP 3	CP 4	Total:	T1 Mark: 88	T1 Mark before: 78	CP 5	CP 6	Mid Year: 86	CP 7	CP 8	T2 Total: 86
Topics:													
1.1: Linear and Quadratic Functions	3				3		3			3			4
1.2: Basic Classes of Functions	2.5	3.5			3		2.5			3			4
2.1: Limits, Rate of Change, Tangent Lines													
2.2: Limits, a Numerical and Graphical Approach		3	4		3.5		3			3.5			3.5
2.3: Basic Limit Laws		3.5	4		4		3.5			4			4
2.4: Limits and Continuity			3.5		3.5		3.5			3.5			3.5
2.5: Evaluating Limits Algebraically			4	3.5	3.5		4			3.5			3.5
2.6: Limits at Infinity			3	3.5	3.5		3			3.5			3.5
2.7: Intermediate Value Theorem			3	4	3.5		3			3.5			3.5
3.1: Definition of the Derivative				3.5				4		4			4
3.2: Derivative as a Function				3				4		4			4
3.3: Product Rule								3.5	3	3	3.5		3.5
3.4: Quotient Rule								3.5	3	3	3.5		3.5
3.5: Higher Derivatives													
3.6: Trigonometric Derivatives								4	2.5	3			3
3.7: Chain Rule								3	2.5	3	3.5		3
3.8: Implicit Differentiation									1		3		3
3.9: Related Rates											3		3
4.1: Linear Approximation											3	4	4
4.2: Extreme Values											4		

Figure 2. Sample Tracking Sheet

By the second half of the year it was curious to see what the students thought about this approach and their relationship with mathematics. Situated in the Grounded Theory, the data was given a chance to develop into self-emerging themes prompted by the questions outlined below via semi-formal interviews. The data was coded recursively like in grounded theory (Glaser & Strauss, 1967, cited in Patton, 2002), looking for themes to emerge without a priori theory to either verify, refute, or otherwise build from. In what follows, I outline the environment and participants, method and data, and discuss emerging themes.

Environment

Two Calculus 12 classes had the implementation of the Check-Point assessment practice. These classes totalled 28 students in Grades 11 and 12. The school offers three levels of calculus in our school: Calculus BC, Calculus AP, and Calculus 12. Typically, the latter is chosen by students who want early exposure to calculus and are planning to take it in their university years. Flipped classroom approach together with the discussion-based learning were the primary vehicles of instruction and day-to-day structure of these classes. Students were expected to come to class prepared to have watched the videos and attempted a series of questions (Sterelyukhin, 2016).

Participants

Four students were selected from the cohort of the two classes to be interviewed. (1) Ethan. Ethan is a student who works hard in class. He was put into an Accelerated Program in Grade 8. In this program students complete Grade 8-10 mathematics curriculum in two years. The selection process for this includes the marks for the first three units of Grade 8 and teacher recommendation. The advantage of being in this cohort is staying one year ahead of their peers in a regular stream. This put Ethan in the position of already completing Pre-Calculus 12 last year. Ethan was selected because he was showing excellent results and participation in class.

(2) Nancy. Nancy always found mathematics challenging and had many issues with the subject throughout her career at the school. One way to help herself that she developed over the years is to pay a very close attention to examples and then mimic her work based on those. Nancy was taking Pre-Calculus 12 concurrently with Calculus 12. Nancy was chosen because she was showing excellent results, and it was particularly interesting to enquire about such a turnaround in her success in a mathematics class.

(3) John. John has always shown great success in his mathematics classes throughout high school. He exhibited a natural aptitude and interest in mathematics. It looked like math came easy to him and he was able to construct meaning for himself to the level that allowed him to be very successful in every math course he took thus far. John has also come from the Accelerated Stream. John was selected because of his excellent results on the Check-Points and insights he was offering during class discussions.

(4) Sam. Sam came to the school in Grade 11. She was not exposed to flipped classroom and discussion-based learning before. She was in the Accelerated Program at a different school with the same outcome of finishing Grade 10 math in her Grade 9 year. Sam was chosen because of her good results, participation and in-depth conversations about learning over the course of the year. In addition, prior to the interview, Sam had written a summary of her 13 years of learning math.

Method and Data

As outlined above, these four students were chosen to conduct a semi-formal interview about their learning experience with mathematics as a whole and particularly this year. As the interest was situated in student experiences with the new implementation of the course, the questions prepared ahead of time were adapted from a similar Chris McGregor study which dealt with reducing anxiety in a middle-school mathematics classroom (McGregor, 2018):

(1) What does mathematics mean to you?

(2) What does learning of mathematics mean to you?

(3) What has changed for you this year in math? How do you feel you are different in and with math this year?

(4) How are you feeling about learning math this year?

(5) Describe feelings, emotions, associations that come to mind when you are in a math class. Try to reflect on the whole experience.

Interviews took part during the school day when students either had a spare block or lunch period. A quiet place was found without anybody listening in or distracting. All the interviews were recorded, totalling in over one hour of recording time. After the attempt to transcribe the entire collection of recordings and running into timing constraints, it was decided to listen to the interviews first to see if there were any emerging themes from what was heard. After listening to all the recordings, five themes were identified that emerged from careful listening and reflecting. Then, only the excerpts that corresponded to these themes were transcribed. The focus was in what the students were saying and not the aesthetics of speech, pauses, etc. Therefore, other aspects of the recordings were not coded. The following five themes emerged: *Math vs. English, Coming Back to Topics (Using Check-Points), Social Aspect (Not getting it but the rest did), Negative Experiences From the Past, and Enjoying Calculus This Year*.

Due to the constraints in the length of this paper, we only present two out of the five themes here. Below is the data created with the themes heading each set of transcriptions:

Theme 2: Coming Back to Topics (Using Check-Points)

(2) Nancy:
1 T: Do you have explanations for why particularly this year, particularly this is, do you feel any different, do you, like, what's...?
2 N: I think, well, for Calc I like how I can, you give us a second chance. A lot of the time in most of our courses I don't get a second chance, so I'm one of those people if I get something wrong, I want to prove myself I can get to do better. I think it's just also the way how you teach now, how I like it. But I think if every other course was like that, it would really help me to improve, which is nice.
(3) John:
1 T: Anything else you want to mention that you have not from what we have been talking about?
2 J: Just any final statement?
3 T: Yeah.
4 J: Ahm, I think the check-points have been a big thing for me this year because, like I said earlier, I get time to finish it, but I think it is also nice to go over a concept multiple times, especially for learning purposes, I think it's great, cause there is a lot of times where you spend a whole month working on something and then you write one test on it and then it's just, it's gone, the concept does not re-appear until the final exam and then you, crap the bed on it, cause you have not seen it in forever and it's, makes it quite difficult, that I think that sort of approach which I think is nice about the check-point, because you see it a couple of times at least before the end of the year, so I think it's quite effective for the learning purposes, and also the marks.
(4) Sam:
1 S: Oh, also, another thing that I feel like I appreciated this year was the whole "check-point" method because I know that last year we talked about aiming for mastery, how can you preach that thing, but not actually do it, because when we do old system, you are not aiming for mastery,

taking the test, done, that's it, learning something new and that's it. And with the Check-point system you are learning something seeing oh, lol, ok I got this part wrong, I can aim for a "4" next time you try it again and again, until you get a "4". I like that you let us show that we can understand it.

Theme 5: Enjoying Calculus This Year

(1) Ethan:
1 T: Thinking about this year in particular, has anything change for you in terms of math this year, or has it been all kind of the same?
2 E: I don't know if it is necessarily cause like the accelerated program, not being in it now basically, being with my own peers, it definitely makes more sense this year, I'm able to understand it this year, the thing also is that cause having you as my teacher made it much more enjoyable, Pre-Calc 12 was my least favorite year of math. I seem to understand this stuff, and you can sort of picture it a bit more, compared to past years.
(2) Nancy:
1 N: I would say this year it's kinda the only year that I've actually really enjoyed math. So, which is really interesting I don't know, I find it interesting, so I do take Calc I guess, and Pre-Calc 12. And I take Chemistry, English, and Physics, but out of the like 5 courses I take, I enjoy going to math the most. It is interesting, it all kinda turned. No, I'm just saying that not because you are asking me, but over this year I kinda liked it better. I don't really know exactly what it means to me though, but I did really enjoy it.
2 N: I think... over the years I just kind of kept pushing and kept wanting to do better and then I've noticed if I look at my grade 11 marks to my grade 12 marks, I see like a huge jump in progress and so I do go to tutoring, but I actually go to tutoring for Pre-Calculus and I don't go for Calculus, which is also kinda crazy cause I'm doing better in Calc then in Pre-Calc so I find it like always my tutoring actually helping me and so now, where I am at in when I'm in math today I feel more confident, like, I can go to the board and not be nervous I used to find it very nerve-wracking or talking even sometimes I still do if I don't understand a concept, but now if I get it I don't feel as nervous to like express my ideas.
(3) John:
1 T: Describe the feeling, emotions and associations that come to mind when you are in a math class. Try to reflect on your whole experience, not just this year, but if you were to think about your experience of learning mathematics as a student from grade 1 to now when somebody says "this is math now", what do you feel about it?
2 J: Ahm, I think... In the past I had a little of an embarrassed attitude towards it because I had so much pressure on myself with math in particular, I've always, math has always been, supposed to be, my strongest class, so the pressure was to perform and show that I was not less capable than my peers so I wanted to make sure I always was on top of, putting a little pressure in the back of my head, maybe I did not always follow through studying that I should have to maintain what I wanted, but I've always held math in great priority, compared, especially my other subjects, so I think I've always put a lot of pressure on myself in past years. And I think

this year it's been easier knowing that I have a math course under my belt already, with an ok mark, that I can submit already to university, so that's good, but this year it has been a lot less intense feeling.

Themes and Analysis

In this section we will elaborate on the observations from the data on two of the five themes. It was very pleasing that these five themes emerged so clearly from nearly all the students interviewed as the interviews were only about 15 minutes long on average. Furthermore, the themes did not directly follow from the questions that were asked.

First, we turn our attention to the Check-Point method and the opinions about it. Clearly, for the three students who decided to talk about it, Check-Points made a difference. Nancy, John and Sam all comment on the positive aspects of coming back to topics, being given another chance and maximising the learning. Interestingly, that even though their motivations are quite different (Nancy and John are much focused on the marks and measure of achievement through that, and Sam is centered around understanding), the idea of re-visiting, solidifying, getting rid of the “once and only once” moments has given rise to positive experience in a math class when talking about assessment and evaluation. From this feedback we are confident that our idea to make the assessment process a positive experience has succeeded and clearly is making a difference in not only students’ perception of mathematics class, but also in their learning and how they approach it. This echoes McTighe and O'Connor (2005) as it is now widely agreed that the primary purpose of assessment should be to further student learning.

For the last theme of enjoying the class this year one can easily identify the element of less stress and anxiety. Students are telling us that they are more confident in their math learning, feeling that they know it well. They have evidence for such conclusions and are able to track it at all times. This emphasizes the idea that increasing feedback improves achievement (Guskey & Bailey, 2001). Also, knowing that there will be other opportunities to demonstrate their learning along the way decreases the value and the “now and only now” feeling when major assessments are happening. The above connects well with Romagnano (2001) where he dispels a central myth in mathematics assessment, stating that “objectivity would be wonderful if we could have it, but it does not exist” (p. 31). Lastly, assessment should create data of what a learner knows up to the moment of time when the assessment is taking place and feedback from it should prompt a learner to analyse where improvements are needed and to go ahead and make an appropriate change. This enables more opportunity for positive experiences in a math class, and thus the want to keep going, coming back and persevering are more likely to happen, increasing mastery and personal satisfaction from the learning process.

Conclusion

As the process of literature search continued, there were many instances of implementation of different assessment ideas. However, it is evident that assessment practices are influenced by competing philosophies that any single assessment decision is often based on a sometimes inconsistent mix of the various influences (Frey & Schmitt, 2007). There is a common direction to move away from the traditional approach of points, percentages and grades, as McTighe and O'Connor state:

> Too many educators consider [marks] and scores as feedback when, in fact, they fail the specificity test. Pinning a letter (B-) or a number (82%) on a student's work is no more helpful than such comments as "Nice job" or "You can do better." Although good [marks] and positive remarks may feel good, they do not advance learning (McTighe & O'Connor, 2005, p. 16).

From what we have seen in the data, the study of personal relationship with mathematics and learning of mathematics connects directly with assessment. Students are more than willing to share their experiences with the subject and are very honest when talking about their feelings and emotions. It is very interesting to note that these ideas are centered around assessment as it came up in all the interviews, clearly indicating that there are a number of items to be investigated further. One factor seems to be prevailing from all of this: students need to be asked about their learning of mathematics, what they like, what they do not like, where are their positive moments and where there are negative ones. From a small sample of four students from a pool of 28 between the two classes it is evident that their relationship with the learning of mathematics is much more than just marks and tests.

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